ANALYTICAL MODELS ON ACCOUNTING INFORMATION: A MATHEMATICAL SIGNALS APPROACH

Nidhish Kumar Mishra, Saudi Electronic University
Ijaz Ali, Fahad Bin Sultan University
Ashraf Imam, IIMEDU
Asma Khatoon, Imam Abdulrahman Bin Faisal University
Odunayo Magret Olarewaju, Durban University of Technology
Imran Ahmad Khan, Creative Heads Consultants
Asif Baig, Jubail University College

ABSTRACT

In line with the idea of "transaction cost economics," which views markets and organizations as alternative mechanisms for resource allocation, the "numerical signal perspective" refers to our view of the function of accounting in which, in contrast to market prices, which are the typical information medium for resource allocation (i.e., numerical signals), accounting figures (especially accounting profits) produced by the accounting system act as an information medium for coordinating business-related transactions in a way that complements or replaces market prices. The purpose of this paper is to explore a model with desirable features and attributes that are compatible with the development of an analytical model of this "number signal perspective" for analytical research. Therefore, this paper reviews and summarizes the past results of disclosure models in the field of financial accounting, borrowing from the discussion in Verrecchia (2001) and Stocken (2012), which are survey papers on accounting disclosure models. One of the conclusions of this paper is that a "noisy rational expectations type model" that incorporates a certain discretionary nature of accounting information and maintains its concreteness, or a "signal-jamming model" with a certain endogenization of capital market processes, is desirable for the "numerical signal perspective".

Keywords: Accounting Disclosure Model, Numerical Signal Perspective, Noisy Rational Expectations Model, Signal-Jamming Model

INTRODUCTION

Where is the Problem?

In accounting research, as in other adjacent economic sciences, analytical accounting research or mathematical model analysis of accounting (hereinafter referred to as "model analysis" or "analytical model") is one of the major analytical tools. Several model analyses have been developed in accounting journals to date, and more will be developed in the future as well, either by deepening existing discussions or by bringing mathematical techniques to bear on new issues.

Even if we limit our interest in accounting research to the field of financial accounting, excluding the fields of management accounting and auditing, there are a variety of analytical models of different types, both large and small, for various issues or themes in financial accounting today.

Citation Information: Mishra, N.K., Ali, I., Imam, A., Khatoon, A., Olarewaju, O.M., Khan, I.A., & Baig, A. (2021). An analysis on accounting disclosure models for modeling the mathematical signals perspective. *Journal of Management Information and Decision Sciences, 24*(S6), 1-31.

Needless to say, the interests of the researchers, which are presented as a research topic or issue, and the individual analytical models used by the researchers to address those interests are closely related to each other. However, let us dare to consider them separately. The two major research issues or themes that analytical accounting research has addressed so far seem to be "accounting disclosure" and "contracts". It is natural to assume that accounting information is used for various purposes in various economic environments because accounting information constitutes the core of the information environment surrounding enterprises, which is indispensable for various types of enterprises in economic society. As a result, from the perspective of focusing on the function of accounting information in the economy and society, it is not possible to specify a single purpose or function of accounting information, and different or conflicting views will arise. In the world of accounting practice, from the standpoint of concretely shaping accounting standards, it is common to understand what functions accounting information performs or should perform in economic society in two broad categories: "information provision function" as the function that accounting information performs or should perform when the existence of capital markets is in mind, and it has been generally understood that accounting information plays or should play two major functions: "information provision function" as the function that accounting information plays or should play when the existence of capital markets is not specifically considered, and "interest adjustment function" as the function that accounting information plays or should play when the existence of competitive markets in general including capital markets is not specifically considered. Specifically, in the former case, the role of accounting information is assumed to be to provide useful information for investors participating in the capital market to make securities decisions, while in the latter case, the role of accounting information is assumed to be to play in the conclusion and performance of contracts between the company and its stakeholders (customers, suppliers, management, employees, fund providers, government and local governments, etc.) (mainly assuming unlisted companies). In this sense, the role of accounting information in the execution of contracts is assumed. In this sense, the issues or themes in analytical accounting research are the "accounting disclosure" area, which considers the role and function of accounting information with the formation of prices in capital markets in mind, and the "accounting information (accounting figures)" area, which considers the role and function of accounting information in the setting of transaction consideration in various contracts that companies enter into for various transactions without specifically considering the existence of prices in the market. From a slightly different point of view, it should be noted again that in identifying or specifying the purpose and function of accounting information, the position of prices in the market, or more precisely, whether or not the existence of equilibrium prices in a competitive market is kept in mind, is a critically important factor.

Depending on these two areas of interest, accounting disclosure and contracts, analytical models adapted to them will be applied. In this paper, we will limit our scope and focus on the area of "accounting disclosure" and the analytical models used in this area. One of the reasons for this is that, needless to say, the "information provision function" is positioned as the main purpose and function of accounting in the world of accounting systems, and, at least in principle, accounting standards are constructed based on the "information provision function". In addition, a more important reason for us is that model analysis in the area of "contracts" of accounting information (or accounting figures) is relatively simple in its analytical path, leaving aside the complexity of the analytical model used itself. This is because even if there is a transaction price (transaction consideration), it is not given a major role in itself, and only accounting information (or accounting figures, especially accounting profit) acts as the main numerical signal to reconcile the interests of the parties to the transaction, and accounting information naturally becomes the focus of discussion. This is because accounting information is the natural focus of discussion. On the other hand, in the area of "accounting disclosure," the existence of competitive markets such as capital markets or

Citation Information: Mishra, N.K., Ali, I., Imam, A., Khatoon, A., Olarewaju, O.M., Khan, I.A., & Baig, A. (2021). An analysis on accounting disclosure models for modeling the mathematical signals perspective. *Journal of Management Information and Decision Sciences, 24*(S6), 1-31.

product markets is assumed, and especially when well-organized capital markets as assumed by the "information provision function" are assumed as the destination of accounting disclosure, prices (stock prices) that are considered to efficiently aggregate various information are observable. In particular, if a well-organized capital market is assumed to be the destination for accounting disclosure, where the "information function" is assumed, it is assumed that a price (stock price) that is deemed to efficiently aggregate various information is observable and that the equilibrium price first and foremost acts as a numerical signal for resource allocation (perhaps "capital allocation" would be a better term in the case of capital markets). As a result, the function of accounting information (accounting profit and other accounting figures) as a numerical signal, which was clearly under the "contract" domain, is now ambiguous and difficult to grasp. Under the "accounting disclosure" domain, we must consider the function of accounting information based on the function of prices.

However, if we consider that the function of accounting information in the "accounting disclosure" domain, which assumes the function of prices, is to enhance the informational efficiency of prices by improving the quality of accounting information that is incorporated into equilibrium prices, then, as in the model analysis of accounting information in the "contract" domain, the analytical path itself is still relatively simple. The "information provision function" in the world of accounting has naturally positioned accounting information as one of the various types of information incorporated into stock prices as a signal for resource and capital allocation. Of course, the positioning of information as seen in the "information provision function" is not unique to the accounting field. It is a well-known fact that in the world of finance and economics, there has been much interest in the information efficiency of prices as an aggregator of information, and many research results have been accumulated so far.

In contrast to these mainstream arguments, we trace a part of the argument called the "feedback effect" in the field of finance, which states that while a company's expected cash flows are reflected in its stock price, the stock price, which reflects the company's expected cash flows, in turn, affects the company's expected cash flows. In addition, I have been interested in the debate in the accounting field called the "real effects perspective"3 which argues that how accounting measurements and disclosures are made can, in turn, affect the production and investment decisions of companies and, more specifically, the allocation of resources in the economy *via* valuations (*i.e.*, stock prices) in capital markets. These argue that numerical signals in the economy, whether they are prices or accounting information such as accounting profits, reflect and influence the real.

In the setting of corporate decision-making on the one hand and corporate pricing in the capital market, on the other hand, numerical signals such as price (stock price) in the "feedback effect" and accounting profit or cash flow measurement in the "real effects perspective" affect corporate production and investment decisions. In particular, what are the (inherent) functions of price signals and accounting signals in the "process of real effects" from signals to entities, and what is the interaction between the two signals? Our ultimate goal is to find out how the two signals interact with each other. This is because, in line with the "economics of transaction costs," in which markets and organizations are viewed as alternative resource allocation mechanisms, we position accounting information (specifically, accounting profits) in opposition to the resource allocation signal (information medium) of price. This is because we have our view of accounting functions, which we call the "numerical signal perspective", in which accounting information (accounting figures, especially accounting profit) is positioned in opposition to the signal of resource allocation (information medium) of price, and the artificial signal of accounting information (accounting figures, especially accounting profit) plays a function similar to the function of the signal of price in organization-related transactions. To identify the essential functions of accounting information (or accounting figures such as accounting profit) in the economy and society, we believe it is useful and

Citation Information: Mishra, N.K., Ali, I., Imam, A., Khatoon, A., Olarewaju, O.M., Khan, I.A., & Baig, A. (2021). An analysis on accounting disclosure models for modeling the mathematical signals perspective. *Journal of Management Information and Decision Sciences, 24*(S6), 1-31.

necessary to examine them about the functions of prices, in contrast to the functions of equilibrium prices in competitive markets.

As part of the preparatory work toward this goal, the purpose of this paper is to understand the basic types of the major analytical models that have been used in the field of "accounting disclosure" by referring mainly to Verrecchia (2001); Stocken (2012), which are survey papers on accounting disclosure models.

In the analysis of accounting disclosures that is developed under the assumption of the existence of competitive markets and thus the formation and observability of prices that aggregate information, analytical models such as the (noisy) rational expectations equilibrium model, the Kyle model that assumes imperfect competition, the unraveling argument, the signal-jamming model, and the multiple occupation model have been used as basic types of models. To reiterate, from our point of view, the model should satisfy the following conditions: (i) it should be able to characterize the function of accounting disclosure as an information medium based on the function of market prices as an information medium (*i.e.*, numerical signals), and (ii) it should be able to capture the real impact process of accounting disclosure on the real economy.

Verrecchia (2001) classified public disclosure models in capital market settings (up to around 2000) into three main categories: association-based disclosure, discretionary-based disclosure, and efficiency-based disclosure. Relevance-based disclosure research refers to papers that are fundamentally interested in how exogenously given disclosures are associated with and relate to changes or disruptions in the activities of investors competing in capital market settings as agents of maximizing personal utility. The salient feature of papers in this category is that they study the impact of exogenous disclosures on aggregate and cumulative changes in investor behavior through movements in asset equilibrium prices and trading volumes. The second category, "discretion-based disclosure studies," analyzes how managers and/or firms exercise discretion in disclosing information that they may have. The salient feature of papers in this category is that they treat the disclosure as endogenous by considering managers and/or firms with incentives to disclose information that is known to them. Note that such an endogenous treatment of disclosure typically takes place in the context of a (simple) capital market setting where the market is characterized as a single representative consumer of disclosed information. The last category, "efficiency-based disclosure studies," consists of papers that discuss what kinds of disclosure arrangements (or disclosure strategies) are preferred in the absence of prior knowledge of the information, *i.e.*, ex-ante. A salient feature of this category of papers is that they consider unconditional disclosure choices, and this work is typically done in the context of a capital market setting where the behavior of agents maximizing their utility is endogenous.

For these three categories, or also pointed out as follows (Verrecchia, 2001, 99-100) Relevance-based disclosure research studies the relationship between disclosure and capital markets under the assumption that the incentives for disclosure coordination and/or the efficiency of the disclosure are fixed or exogenous. Discretionary-based disclosure studies" bring in incentives for disclosure activities (but typically in the absence of ex-ante considerations). Finally, "efficiency-based disclosure studies" examine unconditional disclosure choices. More straightforwardly, we also point out that "relevance-based disclosure studies" deal with the endogenization of market processes, "discretion-based disclosure studies" with disclosure incentives, and "efficiency-based disclosure studies" with the efficiency of disclosure.

Verrecchia's (2001) classification perspective on the survey of disclosure models focuses on how disclosure itself is handled in each disclosure model (from homogenization to endogenization of disclosure in the model, and then to non-homogenization of disclosure). In other words, (1) "relevance-based disclosure research"-which asks about the economic impact of the disclosure without showing the reasons for its existence (under the assumption that disclosure exists from the beginning) and under the assumption that disclosure is not manipulated by management; (2)

Citation Information: Mishra, N.K., Ali, I., Imam, A., Khatoon, A., Olarewaju, O.M., Khan, I.A., & Baig, A. (2021). An analysis on accounting disclosure models for modeling the mathematical signals perspective. *Journal of Management Information and Decision Sciences, 24*(S6), 1-31.

"discretionary-based disclosure research"-questions the incentives for disclosure, such as under what conditions disclosure is made and what kind of content is disclosed; and (3) "efficiency-based disclosure research"-questions the very reason for the existence of disclosure, *i.e.*, whether the disclosure is necessary for the economy and society in the first place, and even today, the effectiveness of this approach has not faded. However, we are not interested in public disclosures in general, including voluntary disclosures, but rather in accounting information that is mandatory public disclosure information, verifiable information based on audits (generally, truthful disclosures or disclosures selected within the scope of Generally Accepted Accounting Principles (GAAP) that do not require consideration of the possibility of false disclosures). In addition, given our awareness of the problem of accounting information as a signal that is counter posed to price (specifically, accounting profit), Verrecchia's (2001) "efficiency-based disclosure research"4 is a bit distant from our interests. For this reason, we will limit our discussion to "relevance-based disclosure research" and "discretion-based disclosure research" in the following.

In Verrecchia (2001), a simplified model called the "modeling vignette" is presented as needed to clarify the nature and characteristics of the arguments belonging to each of these major categories. In the next section, I will discuss "relevance-based disclosure research" based on this "modeling vignette," and in section, I will trace the discussion of "discretion-based disclosure research," keeping in mind the comparison with the discussion of Stocken (2012), who similarly surveyed the same research area as what Verrecchia (2001) calls "discretion-based disclosure research. A summary is provided in Section.

RELEVANCE-BASED DISCLOSURE RESEARCH

How is disclosure related to or associated with changes or disruptions in the behavior of diverse and competing investors in a capital market setting that assumes the existence of investors who maximize their utility? The "relevance-based disclosure studies" were a category of accounting disclosure models that attempted to analyze this issue by characterizing the impact of disclosure on the cumulative behavior of individual investors at the time of the disclosure event.

A "Sketch Model" of "Relevance-Based Disclosure Research"

After reiterating that the characteristics of aggregate or cumulative behavior that are particularly interesting for "relevance-based disclosure research" involve two aspects: (1) the relationship between disclosure and price changes and (2) the relationship between disclosure and transaction volume (Verrecchia, 2001), presents ten "sketch models". In the following, we will trace Verrecchia's (2001) argument, keeping in mind our interest and trying to capture the characteristics of each "sketch model" as briefly as possible.

A Simple Model of Disclosure Relevance (Model #1)

First, a remarkably stylized "sketch model" of disclosure and price changes will be presented to motivate the discussion that will follow. Since it is the starting point for the rest of the discussion, I will trace its contents in relative detail. Uncertain firm value is assumed to be represented by a normally distributed random variable $\tilde{v}$ with mean μ and precision h. Also, the disclosure is expressed as$\tilde{y} = \tilde{v} + \tilde{\eta}$. Where $\tilde{\eta}$ is a normal distribution with mean 0 and precision n? Two 5 All quotations are from Verrecchia (2001), except for a few passages where we use the phrase "I think --------" to indicate that they are our views or opinions. Some of the symbols have been changed.

Periods exist, where time T-1 is the period immediately before the disclosure is made and time T is the period immediately after the disclosure is made. Let the asset prices at time T-1 and

Citation Information: Mishra, N.K., Ali, I., Imam, A., Khatoon, A., Olarewaju, O.M., Khan, I.A., & Baig, A. (2021). An analysis on accounting disclosure models for modeling the mathematical signals perspective. *Journal of Management Information and Decision Sciences, 24*(S6), 1-31.

time T be P_{T-1} andP_T, respectively. We now consider the functional relationship between an exogenous disclosure $\tilde{y}$ and the change in asset price $P_T - P_{T-1}$ at time T. The asset price in time T shall be expressed as follows.

$$\tilde{P}_T - \tilde{P}_{T-1} = \alpha + \beta(\tilde{y} - \mu) + \gamma\widetilde{\Omega} + \tilde{\xi}$$

where α, β, and γ are (fixed) parameters, $\widetilde{\Omega}$ represents variables other than $\tilde{y}$ that are related to firm value and price changes, and $\tilde{\xi}$ represents variables that are not related to firm value (*i.e.*, noise). Here, the coefficient β on $\tilde{y}$ can be interpreted as an element of the functional relationship in the price change that results directly from the disclosure, unlike other factors. In the following discussion, if the price change is assumed to take the form of a linear function, as in the above equation, we refer to β as the Disclosure Response Coefficient (DRC) for the price change for convenience.

In this model #1, we assume that all investors in the market are risk-neutral and time T-1 has no (private or public) information about the value of the firm. Since there is no information, all expectations in time T-1 are based on unconditional expectations about$\tilde{v}$, which are μ. Furthermore, the price of the asset at time T-1 is $P_{T-1} = \mu$ because the investor is risk neutral. In time T-1, a disclosure is made (*i.e.*, $\tilde{y} = y$ is disclosed). Here, we assume that this disclosure is only information about the enterprise value, or if there is other information about the enterprise value (e.g., private information) that is manifested at the same time, this information is included in $\tilde{y} = y$. That is, concerning the pricing of a firm, $\tilde{y}$ is one sufficient statistic for $\tilde{y}$ and all other information; $\tilde{y} = y$ is a sufficient statistic for all information, and when the investor is risk neutral, it is computed as $P_T = E[\tilde{v}|\tilde{y} = y] = \mu + \frac{n}{h+n}(\tilde{y} - \mu)$, and the resulting change in price is shown as

$$\tilde{P}_T - \tilde{P}_{T-1} = \frac{n}{h+n}(\tilde{y} - \mu)$$

Here, expression $\tilde{y} - \mu$ can be interpreted as "disclosure surprise" because it represents the extent to which disclosure μ deviates from$\frac{n}{h+n}$, which is also the expected value of corporate value $\tilde{v}$. Expression $\tilde{y} = y$ represents the ratio of the accuracy of disclosure n to the overall accuracy of corporate value $h + n$, which is conditional on disclosure. Also, since it is$Var[\tilde{P}_T - \tilde{P}_{T-1} | \tilde{y} = y] = 0$, all scatter in price changes is explained by the disclosure in time T.

The model is quite elegant in the sense that it shows the impact of disclosure in the market simply and straightforwardly, but this elegance is achieved at the expense of an extreme stylization of how the market works. In this model, there is no relevant information about corporate value other than that which arises directly from disclosure. Perhaps more importantly, the model describes a world in which no transactions occur. The reason is that beliefs are homogeneous in both time T-1 and time T, and hence there is no informed basis for trading. Thus, if the minimal condition for "robustness of the model" is that at the time of disclosure, a certain amount of trading occurs, and then more work is needed. There seems to be a need to appeal to certain elements of investor diversity for transactions to take place. This is because transactions are fundamentally the result of differences among investors. For example, differences in opinion, differences in endowment, differences in the use of information by investors, and so on. As a result, in any theory-based characterization of the interaction of individual utility-maximizing agents competing in capital markets, the following are the attributes of investor rationality and diversity that seem important for the model to incorporate, or at least for the model to address These attributes are listed in turn below. These attributes, in turn, will be incorporated into the following "sketch model".

1. Investors have a wide variety of information.
2. Investors make rational inferences from market prices.
3. Investors reasonably anticipate disclosure.
4. In addition to possessing a variety of information, investors likewise possess information of varying or heterogeneous quality.
5. Investors interpret disclosures in a variety of ways.

Citation Information: Mishra, N.K., Ali, I., Imam, A., Khatoon, A., Olarewaju, O.M., Khan, I.A., & Baig, A. (2021). An analysis on accounting disclosure models for modeling the mathematical signals perspective. *Journal of Management Information and Decision Sciences, 24*(S6), 1-31.

6. Investors incorporate disclosures into their beliefs in a variety of ways. That is, some agents behave differently from Bayesian behavior (in the narrow sense) in how they incorporate disclosure into their posterior expectations.
7. Investors condition their beliefs on a variety of stimuli. That is, specifically, investors make rational inferences from both market prices and trading volumes.

Investors with Diverse Information (Model #2)

Consider an economy with two assets, a safe asset (New Melaire) and a risky asset (or simply "asset" if there is no confusion). For example, assume that there are a large number of investors, say N people, and that each investor *i* holds a risky asset x_i and a safe asset b_i. For convenience, let x denote the per capita supply of risky assets. Here, x is defined as $x = \Sigma_i \left(\frac{x_i}{N}\right)$. In addition, it is useful to accept the fact that the supply of risky assets per capita $\tilde{x}$ is normally distributed with mean 0 and precision *t*. When examining the variability of price changes resulting from disclosure in the following "sketch model," we interpret $\tilde{x}$ as noise and t as the precision of the noise. As in the previous model #1, it is assumed that there is no information about risky assets in time T-1. The result is $P_{T-1} = \mu$, as in Model #1. However, before a transaction takes place in time T, each investor i acquires different private information about the value of $\tilde{v}$. Here, the private information to be acquired is denoted by $\tilde{z}_i = \tilde{v} + \varepsilon_i$, and $\tilde{\varepsilon}_i$ is assumed to be normally distributed with mean 0 and precision s. Parameter $\tilde{\varepsilon}_i$ is the "noise" term. Needless to say, $\tilde{\varepsilon}_i$ captures the extent to which each investor's information about uncertain asset values is accurate. For example, a large s represents highly accurate private information, while a small s represents highly inaccurate private information. For convenience, in the following, we assume that the covariance of all combinations of error terms (or noise terms) is zero. That is, for example,$E[\tilde{\eta}\,\tilde{\varepsilon}_i] = E\left[\tilde{\varepsilon}_i\tilde{\varepsilon}_j\right] = 0$.This implies that $\tilde{v}$, $\tilde{y}$ and $\tilde{z}_i$ are trivariate normal distributions with mean (μ, μ, μ) and covariance matrix as follows.

$$\begin{bmatrix} h^{-1} & h^{-1} & h^{-1} \\ h^{-1} & h^{-1}+n^{-1} & h^{-1} \\ h^{-1} & h^{-1} & h^{-1}+s^{-1} \end{bmatrix}$$

As a result, when investors condition their expectations on public information and their private information, their expectations can be expressed as follows

$$E[\tilde{v}|y, z_i] = \frac{h\mu + ny + sz_i}{h+n+s}$$

In addition, the accuracy of investors' expectations can be shown by the following equation

$$(Var[\tilde{v}|y, z_i])^{-1} = h + n + s$$

Furthermore, $\tilde{\varepsilon}_i's$ is assumed to have a finite variance. From this assumption, by the law of large numbers, for all realizations of $\tilde{\varepsilon}_i's$, we have $lim_{N\to\infty}\left(\frac{1}{N}\right)\Sigma_i\varepsilon_i \to 0$. This implies that for any given realization of $\tilde{z}_i's$, it is $lim_{N\to\infty}\left(\frac{1}{N}\right)\Sigma_i z_i \to v$.

When the motivation for extending Model #1 is to ensure trading volume, it is useful to relax our assumption that all investors are risk-neutral. If all investors are risk-neutral and hold different private information, trading volume may still occur, but in this case it will be of a highly stylized nature. The investor with the highest conditional expectation of the value of the asset at time T (*i.e.*, the investor with the highest $E[\tilde{v}|y, z_i]$ will capture the aggregate supply of the asset at a minimum. Thus, under utility for quantity g of the consumption good given by $U(g) = -exp\,[-\frac{g}{r}]$, the investor agent is assumed to be risk averse. Here, r measures the investor's tolerance for risk. This utility function is a (negative) exponential function and has desirable attributes that make sense for a utility function. In other words, this utility function is an increasing function of g and is concave, which implies that investors prefer more over fewer consumption

Citation Information: Mishra, N.K., Ali, I., Imam, A., Khatoon, A., Olarewaju, O.M., Khan, I.A., & Baig, A. (2021). An analysis on accounting disclosure models for modeling the mathematical signals perspective. *Journal of Management Information and Decision Sciences, 24*(S6), 1-31.

goods, but the increase is diminishing. However, the real attraction of the negative exponential function is that, when it is used in conjunction with the normal distribution, it results in easier analysis.

The next step is to determine P_T. To do this, we assume that P_T is generated from perfect competition due to the existence of a large number of investors. Investors under perfect competition behave in the market as if their actions have no effect on prices, and in equilibrium, this assumption is assumed to be true. In characterizing transactions on a theoretical basis, perfect competition is achieved by assuming that the number of investors in the market is large (typically, countably infinite). This ensures that, at the aggregate level, it reflects the combined decisions of all market participants, while being sufficiently atomic that the market behavior of individual agents does not have a noticeable effect on prices. From all points of view, perfect competition seems to be a reasonable assumption about deep markets and/or about widely traded assets. In addition, one of the reasons why perfect competition is a desirable vehicle for "relevance-based disclosure studies" is that it makes the "game" played by market participants in determining market equilibrium prices considerably simpler, which is sufficient here. That is, the fact that individual investors can ignore the effect of their actions on prices simplifies the determination of the equilibrium price to a large extent, especially if we assume that transactions take place over some time (to be discussed later).

Let's consider a Walrasian equilibrium model (Walrasian equilibrium) in conjunction with perfect competition. Walras's view of how market-clearing prices are determined in the markets in which divisible assets (e.g., corporate stock) are exchanged is that, first, investors present demand curves for their assets to benevolent and altruistic market makers (usually called "Walrasian auctioneers"), and, second, that the market-clearing price is determined by the demand for the asset. The investor demand curve represents their demand as a function of the price of the asset in question. Based on this information, the Walrasian Auctioneer determines the price at which the aggregate demand for the asset (*i.e.*, the aggregate demand curve of the individual investors) equals the aggregate supply. This price "clears the market" and hence represents the equilibrium price.

Now, conditional on the investor's private information z_i, consider the demand of investor $i's$ for a risky asset versus a safe asset whose value remains fixed at 1. The demand for the risky asset is D_i and the demand for the safe asset is B_i. The price at which the former risky asset is traded is P_T, and the price at which the latter safe asset is traded is 1. Thus, investor i's initial portfolio holding is $x_i P_T + b_i$. The cost of holding the portfolio represented by $D_i$and $B_i$is $D_i P_T + B_i$, and the return earned by holding that portfolio is $D_i v + B_i$ Taken together, this means that the net return on holding the portfolio (and the net amount of income from the value of the initial portfolio holdings of investor i's), represented by D_i and B_i, is $D_i(v - P_T) + x_i P_T + b_i$. The expected value of this portfolio for investor i, based on his private information z̃_i and y, is $E[U(D_i(\tilde{v} - P_T) + x_i P_T + b_i)|y, z_i]$.

To determine the value of P_T, we must first calculate each investor's demand for D_i and B_i. When a negative exponential utility function is used in conjunction with a normal distribution, we obtain the following result, which is expressed as a linear expression of the exponential variable. In other words:

$$E[U(D_i(\tilde{v} - P_T) + x_i P_T + b_i)|y, z_i]$$
$$= -exp\left[-\left(\frac{1}{r} D_i E[\tilde{v}|y, z_i, q] - \frac{1}{2r^2} D_i^2 Var[\tilde{v}|y, z_i, q] - \frac{1}{r} D_i P_T + \frac{1}{r} x_i P_T + \frac{1}{r} b_i\right)\right]$$

In determining its optimal portfolio, each investor will choose $D_i$that maximizes the above equation. As a result, we obtain the following equation

$$D_i = r \frac{E[\tilde{v}|Y, z_i] - P_T}{Var[\tilde{v}|y, z_i]}$$

This is the standard demand equation resulting from a negative exponential function tied to a normal distribution. This implies that the demand for an asset is equal to the investor's expectation

Citation Information: Mishra, N.K., Ali, I., Imam, A., Khatoon, A., Olarewaju, O.M., Khan, I.A., & Baig, A. (2021). An analysis on accounting disclosure models for modeling the mathematical signals perspective. *Journal of Management Information and Decision Sciences, 24*(S6), 1-31.

of the asset's value conditional on his private information and disclosures, less the asset's price, adjusted by (1) his tolerance for risk (*i.e.,* r) and (2) the investor's confidence in his posterior expectations (*i.e.,* the denominator), $Var[\tilde{v}|y, z_i]$). From simple arithmetic from multivariate normality, it is $E[\tilde{v}|Y, z_i] = \mu + \left[\frac{n}{(h+n+s)}\right](y-\mu) + \left[\frac{s}{(h+n+s)}\right](z_i - \mu)$and $Var[\tilde{v}|y, z_i] = \frac{1}{(h+n+s)}$. As a result, D_i can be rewritten as

$$D_i = r(h\mu + ny + sz_i - (h+n+s)P_T)$$

The goal continues to be to endogenize P_T. P_T is determined by assuming that the per capita demand for risky assets equals the per capita supply. The demand and supply per capita are equal, *i.e.*, P_T, which achieves $\Sigma_i\left(\frac{x_i}{N}\right) = \Sigma_i\left(\frac{D_i}{N}\right)$, is as follows

$$\tilde{P}_T = \frac{1}{h+n+s}\left(h\mu + n\tilde{y} + s\, lim_{N\to\infty}\frac{1}{N}\Sigma_i z_i - \frac{1}{r}\tilde{x}\right)$$

$$= \frac{1}{h+n+s}\left(h\mu + n\tilde{y} + s\tilde{v} - \frac{1}{r}\tilde{x}\right)$$

As a result, the following equation is obtained.

$$\tilde{P}_T - \tilde{p}_{T-1} = \frac{1}{h+n+s}\left(n(\tilde{y}-\mu) + s(\tilde{v}-\mu) - \frac{1}{r}\tilde{x}\right)$$

Note that these are $E[\tilde{P}_T] = \mu$ and $E[\tilde{P}_T - \tilde{P}_{T-1}] = 0$. One interpretation of $\tilde{P}_T - \tilde{P}_{T-1}$ is that $\tilde{P}_T - \tilde{P}_{T-1}$ represents the change in expectations of $\tilde{v}$ averaged across all investors. Here, changes in expectations are (1) adjusted by the ex post precision of investors' expectations based on their knowledge of y and z_i, and (2) further adjusted by the supply of risky assets per capita (simultaneously adjusted by investors' risk tolerance r). To attract risk-averse investors (assuming that the realized value of the per capita supply of assets, *i.e.,* x, is positive), asset price P_T at time T must fall below the expected value of risky assets, μ (*i.e.,* $E\left[\frac{1}{(h+n+s)}(h\mu + n\tilde{y} + s\tilde{v})\right] = \mu$). We can consider "supply adjustment," $-[\frac{1}{r(h+n+s)}]\tilde{x}$, as an indication of the extent/degree of the decline. For example, when the investor's risk tolerance is significantly high, implying that the investor is almost risk neutral, it is $r \to \infty$, and the adjustment by the investor's risk tolerance is 0. Similarly, when the accuracy of the investor's ex post expectations is extremely high, that is, when the value of the asset is almost certain for the investor, it is $h+n+s \to \infty$, and again, the adjustment by the accuracy of the ex post expectations is 0.

Rational Inference from Market Prices (Model #3)

Although Walrasian views on perfect competition provide a lot of insight into the price formation process, they may nevertheless be pointed out as conceptually flawed. That is, the implicit assumption in Walrasian equilibrium is the view that investors' beliefs about what a risky asset is worth, or what the value of a risky asset is, are constant or invariant to the price at which the market will clear. This is often referred to as the "exogenous beliefs" model. The conceptual flaw in the "exogenous beliefs" model is that when investors can submit a perfect (ENTIRE) demand curve to a single auctioneer, they should also be able to submit a demand curve based on their expectations of the value of the asset as a function of the market-clearing price. In other words, when investors' demand is a function of price, their beliefs may also be a function of price, and if so, their beliefs may affect their demand. Market equilibrium, in which investors condition their expectations on the price at which the market will clear, is called the "rational expectations" model of trading.

One intuitive way to distinguish the Walrasian model from the rational expectations model of trading is to first imagine the pricing process under Walras. There, the investor determines his demand for an asset-based on (1) his risk tolerance, (2) information about the asset's value, and (3)

Citation Information: Mishra, N.K., Ali, I., Imam, A., Khatoon, A., Olarewaju, O.M., Khan, I.A., & Baig, A. (2021). An analysis on accounting disclosure models for modeling the mathematical signals perspective. *Journal of Management Information and Decision Sciences, 24*(S6), 1-31.

other characteristics of his preferences. The investor then presents his demand curve to a single auctioneer who determines a price that balances the supply of the asset with the aggregated demand. Let us assume that the auctioneer shouts out a market-clearing price that he determines. In Walras, this price would be the price at which the transaction would take place, and nothing more should occur. On the other hand, in the rational expectations equilibrium, the investor should start with the complaint, "Well, if I had known in advance that the market-clearing price was the price that would eventually be shouted, then I would have changed my belief accordingly, and would have presented a different demand curve". Perhaps this complaint/dissatisfaction will cause the equilibrium to be withdrawn, and the auctioneer will unavoidably allow the investor to present the second round of demand curves based on the investor's revised beliefs. Now let's imagine that a different market-clearing price is called for, and again, if the investor knows that this revised price is the market-clearing price, the investor will be presented with yet another set of different demand curves. This will be repeated until the auctioneer finally shouts a certain price and no investor wants to re-sign at that price (*i.e.*, until the investor's complaint/dissatisfaction stops).

The price at which the investor has no further interest in re-signing is the rational expectations market-clearing price. To put it a little differently, in a Walrasian setting, the market-clearing price of an asset is a function of the investor's expectations, but not vice versa, whereas, in rational expectations equilibrium, prices are a function of expectations and expectations is a function of prices. The use of the term "rational expectations" to describe models in the rational expectations literature, as a modeling innovation, is not without risk of misleading the reader into thinking that these models simply introduce the requirement that investors condition their expectations on market-clearing prices. Perhaps, as an alternative, this study should be called a "price-conditioned" transaction model.

The saddle-switching from Walrasian to rational expectations models of trading requires some additional analysis. Among other things, a key feature of the rational expectations equilibrium is that investors infer that the market-clearing price of an asset contains information about the asset's value. As a result, if investors condition their expectations on price, in addition to their private information, they will pick up more insight into the value of an uncertain asset than if they had ignored price.

Let us now assume, in addition to carrying over all the assumptions introduced earlier, that the investor supposes that the market equilibrium price at time T takes the form

$$\tilde{P}_T = a + b\tilde{v} + c\tilde{y} - d\tilde{x}_i$$

Where a, b, c, and d are fixed parameters. Let q̃ be defined as follows.

$$\tilde{q} = \frac{\tilde{P}_T - a - c\tilde{y}}{b} = \tilde{v} - \frac{d}{b}\tilde{x}$$

The variable $\tilde{q}$ represents the additional information that the investor picks up from the price by manipulating the price to produce essential information about $\tilde{v}$. If the investor uses $\tilde{q}$, along with $\tilde{v}$, $\tilde{y}$, and $\tilde{z}_i$, then the four-variate normal distribution has mean (μ,μ,μ,μ) and covariance matrix given by

$$\begin{bmatrix} h^{-1} & h^{-1} & h^{-1} & h^{-1} \\ h^{-1} & h^{-1}+n^{-1} & h^{-1} & h^{-1} \\ h^{-1} & h^{-1} & h^{-1}+s^{-1} & h^{-1} \\ h^{-1} & h^{-1} & h^{-1} & h^{-1}+\left(\frac{d}{b}\right)^2 t^{-1} \end{bmatrix}$$

As a result, when investors condition their expectations on (1) disclosure, (2) their private information, and (3) the price (through q̃) as an additional source of information, their expectations are shown as

Citation Information: Mishra, N.K., Ali, I., Imam, A., Khatoon, A., Olarewaju, O.M., Khan, I.A., & Baig, A. (2021). An analysis on accounting disclosure models for modeling the mathematical signals perspective. *Journal of Management Information and Decision Sciences, 24*(S6), 1-31.

$$\text{E } E[\tilde{v}|y, z_i, q] = \frac{h\mu+ny+sz_i+\left(\frac{b}{d}\right)^2 tq}{h+n+s+\left(\frac{b}{d}\right)^2 t}$$

In addition, the accuracy of investors' expectations can be shown by the following equation

$$(Var[\tilde{v}|y, z_i, q])^{-1} = h + n + s + \left(\frac{b}{d}\right)^2 t$$

To determine the value of P_T, again, we must first calculate the demand of each investor for D_i. As before, the negative exponential utility function produces the following result, which is linear in the exponential variable

$$E[U(D_i(\tilde{v} - P_T) + x_i P_T + b_i)|y, z_i, q]$$
$$= -exp\left[-\left(\frac{1}{r} D_i E[\tilde{v}|y, z_i, q] - \frac{1}{2r^2} D_i^2 Var[\tilde{v}|y, z_i, q] - \frac{1}{r} D_i P_T + \frac{1}{r} x_i P_T + \frac{1}{r} b_i\right)\right]$$

In determining its optimal portfolio, each investor chooses the D_i that maximizes the above equation. This yields the following equation

D

$$D_i = r\frac{E[\tilde{v}|y,z_i,q]-P_T}{Var[\tilde{v}|y,z_i,q]}$$

This demand equation has the same expression as the previous one, except for the fact that the investor conditions his expectations on the price (through q) in addition to y andz_i. As a result, D_i can be rewritten as

$$D_i = r\left\{h\mu + ny + sz_i + \left(\frac{b}{d}\right)^2 tq - \left(h + n + s + \left(\frac{b}{d}\right)^2 t\right) P_T\right\}$$

As before, we endogenize $P_T$by setting the per capita supply of risky assets equal to the per capita demand, or in other words, by setting $x = \Sigma_i\left(\frac{x_i}{N}\right) = \Sigma_i\left(\frac{D_i}{N}\right)$. The value of P_T resulting from this endogenization process is as follows

$$\tilde{P}_T = \frac{1}{h\mu+ny+sz_i+\left(\frac{b}{d}\right)^2 t}\left(h\mu + n\tilde{y} + \left(\frac{b}{d}\right)^2 t\tilde{q} + slim_{N\to\infty}\frac{1}{N}\Sigma_i\tilde{z}_i - \frac{1}{r}\tilde{x}\right)$$
$$= \frac{1}{h\mu+ny+sz_i+\left(\frac{b}{d}\right)^2 t}\left(h\mu + n\tilde{y} + \left(s + \left(\frac{b}{d}\right)^2 t\right)\tilde{v} - \left(\frac{1}{r} + \frac{b}{d}t\right)\tilde{x}\right)$$

Let us note that the investor's original conjecture, $\tilde{P}_T = a + b\tilde{v} + c\tilde{y} - d\tilde{x}$, is self-fulfilling (*i.e.*, rational). The result must be as follows.

$$\frac{b}{d} = \frac{s+\left(\frac{b}{d}\right)^2 t}{\frac{1}{r}+\left(\frac{b}{d}\right)t}$$

This implies $\frac{b}{d} = rs$. Hence, the self-fulfilling equilibrium can be characterized by coefficients a, b, c, and d, denoted in the expression $\tilde{P}_T = a + b\tilde{v} + c\tilde{y} - d\tilde{x}$ as follows

$$a = \frac{hm}{(h+n+s+(rs)^2 t)}$$
$$b = \frac{(s+(rs)^2 t)}{(h+n+s+(rs)^2 t)}$$
$$c = \frac{n}{(h+n+s+(rs)^2 t)}$$
$$d = [\left(\frac{1}{r}\right) + \frac{rst}{(h+n+s+(rs)^2 t)}$$

This implies the following equation.

$$\tilde{P}_T - \tilde{P}_{T-1} = \frac{1}{h+n+s+r^2 s^2 t}\left(n(\tilde{y} - \mu) + (s + r^2 s^2 t)(\tilde{v} - \mu) - \left(\frac{1}{r} + rst\right)\tilde{x}\right)$$

Note that this representation of price changes is identical to the previous case, except for the additional information associated with conditioning expectations on prices. The conditioning of expectations on prices produces an additional "information kick" that results in more accurate beliefs in the rational expectations model than in the Walrasian model. Specifically, the accuracy of

Citation Information: Mishra, N.K., Ali, I., Imam, A., Khatoon, A., Olarewaju, O.M., Khan, I.A., & Baig, A. (2021). An analysis on accounting disclosure models for modeling the mathematical signals perspective. *Journal of Management Information and Decision Sciences, 24*(S6), 1-31.

expectations in the rational expectations model is $h + n + s + r^2 s^2 t$, and the accuracy of expectations in the Walrasian model is $h + n + s$. This implies that the information kick is $r^2 s^2 t$.

Before moving on to the next model, I would like to point out the role of the two maintained assumptions: Among the various guesses that investors can make about the market equilibrium price at time T, the "rational expectations" literature maintains the assumption that investors make linear guesses about the form of the market-clearing price function. In other words, it is $\tilde{P}_T = a + b\tilde{v} + c\tilde{y} - d\tilde{x}$. This in no way precludes or rules out the possibility that other nonlinear conjectures lead to self-fulfilling equilibria as well. These alternative conjectures have simply not been studied. This constraint of linear guessing is not unique to the "rational expectations" literature, and in the model of imperfect competition discussed below, linear guessing is likewise assumed for the form of the price function.

Another assumption that holds in Model #3 is that investors have a variety of private information. One model that competes with the one discussed here is that investors can be one of two types. In other words, there are two types of investors: informed investors and uninformed investors who pick up some knowledge by conditioning their beliefs on prices. For the non-informed investor, the price in Model #3 is merely a communicator of information from the informed investor to the uninformed investor. In contrast, the price for information investors in Model #3 is both an aggregator of information and a communicator of the aggregated data in the sense that it aggregates/aggregates the various beliefs of many investors (as embodied in z_i).

Reasonable Expectation of Disclosure (Model #4)

Allowing for rational inference from prices seems to be an obvious improvement on the Walrasian model, with relatively little cost in terms of treatability. On the other hand, there is almost certainly another flaw. The flaw is that, given the market setting described above, there is no prior trading round in which market participants can resolve their differences (e.g., differences in risk appetite, initial holdings, private beliefs) before disclosure. Resolving differences through prior rounds of trading is critically important for relevant studies. This is because, without an ex-ante trading round, most of the other factors that are not relevant to the disclosure will be mixed in with price changes at the time of disclosure. For example, imagine a setting in which market participants enter the market at the beginning of the period to exchange some of their assets, and the market-clearing price is formed based on their demand. In addition, imagine that there is some sort of public disclosure about asset values when they enter the market. In this scenario, prices at the end of the period would be a mix of different risk preferences, different initial holdings of risky assets, and different private information, in addition to disclosures. As a result, it becomes difficult to infer the impact of disclosures on prices and to separate and isolate the impact of disclosures from all the other reasons participants trade.

A way to avoid this problem is to first allow market participants a prior trading round before disclosure and then allow second trading round when disclosure occurs. However, it is important to assume that in the first round of trading, market participants will anticipate disclosure in the second round of trading. The advantage of this approach is that all price changes resulting from the second round of transactions will fully represent the price and the impact of disclosure on price changes. The problem here is that it is technically very difficult to allow for the existence of two trading rounds and at the same time meet the other reasonableness criteria discussed earlier.

To illustrate some of these issues, consider the following proposal. We imagine that the prior trading round of the asset occurs in time T-1 and the disclosure of $\tilde{y} = y$ occur in time T. In the rational expectations model, investors are expected to learn from prices. In other words, they condition their expectations on the price. In two trading rounds, the investor in time T should in principle is able to condition his expectations on the price in both time T-1 and time T. In the

Citation Information: Mishra, N.K., Ali, I., Imam, A., Khatoon, A., Olarewaju, O.M., Khan, I.A., & Baig, A. (2021). An analysis on accounting disclosure models for modeling the mathematical signals perspective. *Journal of Management Information and Decision Sciences, 24*(S6), 1-31.

rational expectations model, the price at time T-1 and the price at time T can be described as the form $\tilde{P}_{T-1} = a_{T-1} + b_{T-1}\tilde{v} - d_{T-1}\tilde{x}$ and the form $\tilde{P}_T = a_T + b_T\tilde{v} + c_T\tilde{y} - d_T\tilde{x}$. In addition, since all fixed parameters (*i.e.*, a_{T-1}, a_T, b_{T-1}, b_T, c_T, d_{T-1}, d_T) are assumed to be common knowledge in the rational expectations model, $\tilde{P}_{T-1}$ and $\tilde{P}_T$represent a single system consisting of two pairs of equations for two unknowns, $\tilde{v}$ and $\tilde{x}$. As a result, when the per capita supply is the same in both time T-1 and time T (*i.e.*, $\tilde{x} = x$ in both time T-1 and time T), either $\tilde{P}_{T-1}$ and $\tilde{P}_T$fully manifest $\tilde{v}$ and $\tilde{x}$, or $\tilde{P}_{T-1}$and $\tilde{P}_T$ are redundant. The former occurs when $\tilde{P}_{T-1}$ and $\tilde{P}_T$ are independent equations, and the latter occurs when $\tilde{P}_{T-1}$ and $\tilde{P}_T$ are dependent equations (*i.e.*, $a_{T-1} = a_T$, $b_{T-1} = b_T$, and $d_{T-1} = d_T$). For example, $\frac{(\tilde{P}_{T-1} - a_{T-1})}{b_{T-1}} = \tilde{v} - \left(\frac{d_{T-1}}{b_{T-1}}\right)\tilde{x}$ and $\frac{(\tilde{P}_T - a_T - c_T\tilde{y})}{b_T} = \tilde{v} - \left(\frac{d_T}{b_T}\right)\tilde{x}$. Thus, when $\frac{d_{T-1}}{b_{T-1}} \neq \frac{d_T}{b_T}$, $\tilde{P}_{T-1}$and $\tilde{P}_T$fully manifest $\tilde{v}$ and $\tilde{x}$, and when $\frac{d_{T-1}}{b_{T-1}} = \frac{d_T}{b_T}$, $\tilde{P}_{T-1}$ and $\tilde{P}_T$are redundant.

While both fully revealing equilibria and price-redundant equilibria may exist, the advantage of focusing exclusively on the latter, relying on investor speculation, is that there is little evidence that prices "fully reveal" asset values in real institutional settings. More importantly, it can be shown that the price-redundancy equilibrium is general equilibrium.

In the context of our assumptions, allowing investors to trade at time T-1 yields the following expression for the price at time T-1.

$$\tilde{P}_{T-1} = \frac{1}{h+s+r^2s^2t}\left(h\mu + (s + r^2s^2t)\tilde{v} - \left(\frac{1}{r} + rst\right)\tilde{x}\right)$$

To digress a bit, this representation of $\tilde{P}_{T-1}$ is very similar to the representation of price in time T in the previous model (Model #3), except for the fact that it does not include disclosure (*i.e.*, $\tilde{y}$). In other words, except for the disclosure, $\tilde{P}_{T-1}$ in this model is the same expression as $\tilde{P}_T$in model #3. Note that despite this identity, $\tilde{P}_T$in Model #3 is the result of an investor acting short-sightedly in the sense of failing to anticipate disclosure in time T, whereas $\tilde{P}_{T-1}$ in this Model #4 relies on an investor evolving endogenously and reasonably anticipating disclosure in time T. It can then be shown that the price of the asset at time T is as follows.

$$\tilde{P}_T = \frac{1}{h+n+s+r^2s^2t}\left(h\mu + n\tilde{y} + (s + r^2s^2t)\tilde{v} - \left(\frac{1}{r} + rst\right)\tilde{v}\right)$$

As a result, after some arithmetic operations, we obtain the following equation.

$$\tilde{P}_t - \tilde{P}_{t-1} = \frac{n}{h+n+s+r^2s^2t}\left(\tilde{y} - \frac{h\mu + (s+r^2s^2t)\tilde{v} - \left(\frac{1}{r}+rst\right)\tilde{x}}{h+s+r^2s^2t}\right)$$

One interpretation of the following expression is that it is a disclosure "surprise" in a price change, and

$$\tilde{y} - \frac{hm + (s+r^2s^2t)\tilde{v} - rst\tilde{x}}{h+s+r^2s^2t}$$

Also, the following expression is “noise”

$$\frac{\left(\frac{1}{r}\right)\tilde{x}}{h+s+r^2s^2t}$$

This is what it is all about.

To digress a bit, the importance of the two periods is that it allows for the study of changes in price movements that are consistent with disclosures (e.g., earnings announcements). Concerning the assumption that the level of noise is the same in both periods (*i.e.*, $\tilde{x} = x$ in time T-1 and time T), recall that $\tilde{x}$ represents a liquidity and/or asset supply shock. Hence, we can interpret this assumption as implying that there is a persistent level of liquidity and/or supply shocks surrounding the earnings announcement (immediately before and after). However, the basic role of this assumption is one of convenience, and it is easily generalized to more complex settings.

Citation Information: Mishra, N.K., Ali, I., Imam, A., Khatoon, A., Olarewaju, O.M., Khan, I.A., & Baig, A. (2021). An analysis on accounting disclosure models for modeling the mathematical signals perspective. *Journal of Management Information and Decision Sciences, 24*(S6), 1-31.

Note that one assumption that has been maintained is that private information is information about the value of the risk asset (*i.e.*, $z_i = \tilde{v} + \tilde{\varepsilon}_i$), not private disclosure expectations (*i.e.*, $\tilde{z}_i = \tilde{y} + \tilde{\varepsilon}_i$).

To allow for the private information expectation of disclosure, adapting the model presented here to it is a simple exercise. However, to maintain continuity in the discussion, we will continue to assume that private information is only information about the value of risky assets in the following discussion.

Private Information of Different Quality (Model #5) and Different Interpretations of Common Disclosures (Model #6)

"The models we've developed so far have many attractive features. Investors have rational expectations in the sense that they condition their expectations on the price, and in the sense that they anticipate disclosure by establishing equilibrium before its dissemination to society. The problem is that there is no transaction volume in time T where disclosure takes place" (Verrecchia, 2001), and after pointing out one flaw in the "sketch model" so far, moving the viewpoint to the relationship between disclosure and transaction volume, as one device to guide the implementation of the transaction in time T where disclosure takes place, in model #5, to relax the assumption maintained in previous "sketch models" that the accuracy of private information held by investors is the same, to overcome the defects of model #5, which suggests that the volume of transactions does not result in a price change in Model #6, assuming that the accuracy of the private information held by the investor is different. The "sketch model" is discussed, which loosens the assumptions maintained in the previous "sketch model" that investors interpret disclosure in a certain common style, and assumes that investors interpret disclosure in various ways.

Heuristic Behavior (Model #7)

In addition to Bayesian rational investors (Bayesian investors), a "sketch model" will be discussed, which relaxes the assumption maintained throughout previous analyses that "all investors in the market follow Bayesian rules and use any information, whether private or public" (Verrecchia, 2001). In addition to the Bayesian rule-compliant rational investors (Bayesian investors), a "sketch model" that assumes the existence of investors who act heuristically will be discussed.

Imperfect Competition (Model #8)

In model #7 above, results were presented that suggest that heuristic behavior would not survive. One of the reasons for such a result is a consequence of the assumption that the market is perfectly competitive, and here the "sketch model" of imperfect competition is considered. Since the discussion is of interest to us, we will trace this "sketch model" in detail below, following Verrechia (2001). One assumption that is maintained throughout the analysis is that markets are efficient. However, markets may not be perfectly competitive if the behavior of certain investors affects the prices at which their trades are conducted. One way to make an institutional case for the possibility of investor behavior influencing prices is to imagine the existence of large institutional investors whose actions could move the market. For example, let us imagine that the market consists of one large institutional investor and a "market" that effectively represents everyone else. For convenience, we will assume below that both the investor (as an institutional investor) and the "market" are both risk-neutral under the utility given by the quantity of the consumption good given by $U(g) = g$.

Citation Information: Mishra, N.K., Ali, I., Imam, A., Khatoon, A., Olarewaju, O.M., Khan, I.A., & Baig, A. (2021). An analysis on accounting disclosure models for modeling the mathematical signals perspective. *Journal of Management Information and Decision Sciences, 24*(S6), 1-31.

The disclosure shall continue to be expressed as $\tilde{y} = \tilde{v} + \tilde{\eta}$. However, in the following, we assume that large institutional investors know $\tilde{\eta} = \eta$. The rationale for knowing is that investors are familiar enough with corporate accounting practices and procedures to understand the set of errors in disclosure that result from random, liberal or conservative accrual accounting. The knowledge of $\tilde{\eta} = \eta$ associated with $\tilde{y}$ implies that the investor knows the value of the company $\tilde{v}$. In contrast, we assume that "the market" is not as perceptive about accounting practices and procedures as investors are, but merely knows $\tilde{y}$.

Imperfect competition implies that investors know that their actions will affect the market price at which their trades will be executed, and they take this into account when they present their demand orders. As a result, investors and "the market" will be playing the following game. First, the investor decides on a demand order that he strongly desires to execute based on his knowledge of $\tilde{v}$. This demand order is then included in the demand order generated by the random shock in asset supply $\tilde{x}$ and is "batched" with. Finally, the "market" executes combined/aggregated demand orders at a single price.

The demand order of the investor shall be denoted by d, the aggregate or total demand order of the investor and random supply shocks by $\widetilde{D} = d + \tilde{x}$, and the price set by the "market" to execute the order by P. (For simplicity of notation, we will drop the subscript "T" when referring to price. As a result, the following model is, in effect, treated exclusively as a one-period trading model). We assume that competition to execute demand orders forces them to be executed at a price that reflects the expected value of the asset conditional on what the "market" knows at the time the order is executed. At the time the demand order is executed, the "market" knows $\tilde{y}$ and $\widetilde{D}$. This implies that it is $P = E[\tilde{v}|\tilde{y}, \widetilde{D}]$. In this game, the first mover is the investor, and hence the investor needs to make certain assumptions about how the "market" will interpret a demand order of a certain size. Here, the investor assumes that the price and total demand order quote set by the "market" based on the disclosures is inferred as follows

$$P = \mu + \beta(y - \mu) + \lambda D$$

In effect, price is a linear function of y and D. Again, the coefficient β is the Disclosure Response Coefficient (DRC), and λ is to be interpreted generally as the depth of the market.

Trading gameplay, through a series of steps in the order of occurrence, can be organized as follows.

(1) Corporate value is realized, which is represented by $\tilde{v} = v$.

(2) Variable $\tilde{y}$=y is disclosed, and the investor observes $\tilde{\eta} = \eta$.

(3) The investor presents a demand order to the "market" and the demand order is associated with a random supply shock, represented by $\tilde{x} = x$.

(4) Based on the aggregate demand order, the "market" sets a price *P* at which trades will be executed (*i.e.*, the "market" chooses P equal to the expected value of the firm conditional on disclosure and aggregate demand). Then, all trades are executed at this price.

(5) The corporation is liquidated and pays a return v to the shareholders.

The equilibrium of the game can be thought of as arising from step (3) and step (4). The individuals who perform steps (3) and (4) act selfishly, respectively. For example, in step (3), the investor determines his demand order by solving the following equation

$$max_d E[v - \tilde{P}|\tilde{v} = v, \tilde{y} = y]$$

Here, we assume that the investor is $\tilde{P} = \mu + \beta(\tilde{y} - \mu) + \lambda\widetilde{D}$. This implies that the investor solves the following equation

$$max_d E[v - \mu - \beta(y - \mu) - \lambda(d + \tilde{x})|\tilde{v} = v, \tilde{y} = y]$$

This then implies the following equation.

$$d = \frac{1}{2\lambda}[v - \mu - \beta(y - \mu)]$$

Citation Information: Mishra, N.K., Ali, I., Imam, A., Khatoon, A., Olarewaju, O.M., Khan, I.A., & Baig, A. (2021). An analysis on accounting disclosure models for modeling the mathematical signals perspective. *Journal of Management Information and Decision Sciences, 24*(S6), 1-31.

As a result of the investor's choice of d, $\tilde{v}$, $\tilde{y}$, and $\widetilde{D} = \tilde{d} + \tilde{x}$ have a trivariate normal distribution with mean (μ,μ,0) and a covariance matrix as follows

$$\begin{bmatrix} h^{-1} & h^{-1} & \frac{1}{2\lambda}h^{-1}(1-\beta) \\ h^{-1} & h^{-1}+n^{-1} & \frac{1}{2\lambda}(h^{-1}-\beta(h^{-1}+n^{-1})) \\ \frac{1}{2\lambda}h^{-1}(1-\beta) & \frac{1}{2\lambda}(h^{-1}-\beta(h^{-1}+n^{-1})) & \frac{1}{4\lambda^2}(h^{-1}-2\beta h^{-1}+\beta^2(h^{-1}+n^{-1})+t^{-1}) \end{bmatrix}$$

In step (4), the "market" sets P conditional on the disclosure and the aggregate demand received.

The covariance matrix given above implies that it results in the following relationship.

$$E[\tilde{v}|\tilde{y},\widetilde{D}] = \mu + \frac{4\lambda^2 n + t\beta}{4\lambda^2 n + t + 4\lambda^2 h}(y-\mu) + \frac{2\lambda t}{4\lambda^2 n + t + 4\lambda^2 h}$$

Note, however, that for investors' initial assumptions about β and λ to be realized, they must be $\beta = \frac{4\lambda^2 n + t\beta}{4\lambda^2 n + t + 4\lambda^2 h}$ and $\lambda = \frac{2\lambda t}{4\lambda^2 n + t + 4\lambda^2 h}$. This would then imply $\beta = \frac{n}{(h+n)}$ and $\lambda = \frac{1}{2}\sqrt{\frac{t}{(h+n)}}$. In summary, a self-fulfilling equilibrium is an equilibrium in which the price at which a demand order is executed is given by

$$\tilde{P} = \mu + \frac{n}{h+n}(\tilde{y}-\mu) + \frac{1}{2}\sqrt{\frac{t}{(h+n)}}\widetilde{D}$$

where $\frac{n}{h+n}$ is the DRC (Disclosure Response Coefficient) and $\frac{1}{2}\sqrt{\frac{t}{h+n}}$ is the depth of the market.

From a disclosure perspective, this equilibrium has several interesting features. First, unlike the perfect competition model, investors do not take infinitely long or infinitely short positions in an asset, even if they know the value of the asset (investors know $\tilde{v} = v$). The reason is that the investor must consider the impact of his demand order on the price at which his demand order would be executed. The larger the investor's demand order (*i.e.*, the larger d), the more the investor will expect it to cost to execute his demand order (*i.e.*, the higher$E[\tilde{P}]$). For example, since $\lambda > 0$, if aggregate demand is positive (*i.e.*, $D > 0$), the investor will undertake a transaction that will be executed at a higher price than the price implied by the "market" (*i.e.*, $P = \mu + [\frac{n}{h+n}](y-\mu)$, knowing only the disclosure y. Another feature of investor demand orders is that, like aggregate demand, they are unrelated to disclosure. In other words, $[(\tilde{y}-\mu)\tilde{d}] = E[(\tilde{y}-\mu)\widetilde{D}] = 0$. The intuition behind this result is that, since $\tilde{y}$ is public information, when an investor presents his demand order d, he knows $\tilde{y}$, and when his demand order is executed, he knows that the information content of $\tilde{y}$ will be fully priced into $\tilde{P}$. As a result, investors will adjust their demand orders to account for the impact of disclosure on prices. This is equivalent to ensuring that the investor's demand order and the disclosure are uncorrelated. Finally, notice that the DRC (disclosure response coefficient) in this model is identical to the DRC that arises in the context of Model #1, which assumes that the only information in the economy is that which arises directly from disclosure. The intuition for this is that the DRC captures the impact of disclosure, while the coefficient of aggregate demand captures the incremental knowledge that comes from observing the aggregate demand D that is added to disclosure.

Revisiting Heuristic Behavior (Model #9)

Based on the previous discussion of Model #7 and Model #8, a "sketch model" suggesting that heuristic investors may survive if heuristic behavior is associated with imperfect competition is

Citation Information: Mishra, N.K., Ali, I., Imam, A., Khatoon, A., Olarewaju, O.M., Khan, I.A., & Baig, A. (2021). An analysis on accounting disclosure models for modeling the mathematical signals perspective. *Journal of Management Information and Decision Sciences, 24*(S6), 1-31.

examined by assuming an economy with two types of large institutional investors, Bayesian and heuristic.

Conditioning of Beliefs on Trading Volume (Model #10)

In discussing the last "sketch model" in "relevance-based disclosure research," Verrecchia (2001, 133) notes, "I consider the role of one assumption that has been maintained to the end. In all of the models discussed so far, agents participating in the market, whether investors or market makers, condition their expectations exclusively on aggregate net demand, either (1) indirectly through market prices (e.g., Models #3-#6) or (2) directly as in "markets" that condition their expectations on total net demand (e.g., Model #8). This raises the question of whether investors and/or "the market" derive benefits from conditioning their expectations on other variables such as trading volume (other than aggregate net demand), and how this changes various market characteristics". With these questions in mind, Model #10 presents a "sketch model" in which the market is assumed to condition on aggregate net demand as well as on aggregate trading volume, in other words, the market is assumed to condition on two sources of information, and the implications of the inferences drawn from conditioning expectations on these two sources of information are examined.

Summary of "Relevance-Based Disclosure Studies

Verrecchia (2001), in his discussion of "relevance-based disclosure research" that assumes disclosure is exogenous, praises it for successfully providing a detailed characterization of the relationship or association between disclosure, price changes, trading volume, and other market phenomena (e.g., market depth) for a broad class of diverse investors. On the other hand, he points out the following issues that were not addressed in the study.

1. The role of various analyst and/or management forecasts before disclosure
2. The role of asymmetric tax effects tied to disclosure
3. Endogenous initial possession of investors' private information
4. Ignoring the incentive to sell and/or distribute the information

In the setting of companies on the one hand and capital markets on the other, if the focus is on accounting information rather than company-related information in general, from the standpoint of the "accounting" disclosure model, it seems natural that the focus will be on the behavior of managers or companies as information providers in terms of information transmission rather than on the behavior of individual investors in capital markets. In this sense, (4) in particular is an interesting point of discussion for us.

Key Components of the "Relevance-Based Disclosure Research" Analytical Model

Based on our view of accounting functions, the "numerical signal perspective," what are the characteristics or attributes of the most appropriate or desirable models for analysis in the area of accounting disclosure? More specifically, from what elements should such a model be constructed? With these questions in mind, I would like to summarize the elements that an analytical model of accounting disclosure should have, borrowing from Ohlson & Buckman's (1980) discussion, which can be regarded as an early perspective paper on analytical models of accounting disclosure. Ohlson & Buckman (1980), a perspective paper at the stage when the information analysis model was finally starting to be developed in the accounting field, may seem rough, but it is also attractive in the sense that it shows great expectations and hopes for future development due to its early stage. Whatever the intentions of Ohlson & Buckman (1980), they emphasize the functioning of capital markets with the participation of investors under the general

Citation Information: Mishra, N.K., Ali, I., Imam, A., Khatoon, A., Olarewaju, O.M., Khan, I.A., & Baig, A. (2021). An analysis on accounting disclosure models for modeling the mathematical signals perspective. *Journal of Management Information and Decision Sciences, 24*(S6), 1-31.

equilibrium model, under the (implicit) assumption that disclosure is made, and the accounting disclosure model that Ohlson & Buckman (1980) have in mind is certainly similar to what Verrecchia (2001) calls "relevance-based disclosure research". Therefore, the main components of the analytical model that they refer to are also the model elements that should be considered in "relevance-based disclosure research," and are also useful for understanding "relevance-based disclosure research". The following is our summary of Ohlson & Buckman (1980) say Financial information (including accounting the representative literature on this issue is, for example, Admati & Pfleiderer (1986, 1988) information) about the outcomes of business firms facilitates and promotes the decision-making process used by rational investors in determining their consumption-investment plans. Here, the use of information in the decision-making process occurs in the context of a market that allocates goods and resources and implicitly allocates "risk". Therefore, it is necessary to recognize that some of the parameters of the investor's decision problem are, in fact, endogenously generated by the simultaneous actions of all investors participating in the market. This would suggest that basic insights in terms of the economic role of the production and dissemination of information must be developed in the context of general equilibrium analysis. This is tantamount to saying that we are interested in the economic mechanisms that allocate commodities and other resources among individuals ("households" or "consumers"), among firms ("producing entities"), by period, etc., and how information affects the final allocation outcomes of these mechanisms. Depending on the nature/property of the mechanism in question, individual preferences, and beliefs, the distribution of initial holdings among individuals, the number and type of commodities, the opportunities created by production technologies, etc., the information will likely have substantially different welfare implications.

In the face of this starkly complex reality, the basic objectives of information welfare analysis have led us to ask what are the essential components or model properties of an analytical model, and what are the attributes of the economic environment that an analytical model must capture that it cannot do without if meaningful conclusions are to be drawn about the role of information. Ohlson & Buckman (1980) point out the following characteristics or attributes that should be minimally taken into account in a general equilibrium model.

Certainty *vs.* Uncertainty in the Economic Environment

In a world of certainty, as is usually thought, there can be no information. This point is clear.

Number of Consumption Points (or Points in Time)

A review of the literature suggests four possibilities. In other words, there are four periods: (1) one period, (2) two periods, (3) three or more but finite periods, and (4) infinite periods. In the analysis of the information challenge, most of the literature has focused on the first two cases (or cases where the period dimension does not make a significant difference). Although the two are generally very similar, it would not be correct to suggest that the conclusions reached in the one-period model would hold for the two-period model as well. While many important and fundamental information issues have been successfully dealt with in a one- or two-period framework, deeper and/or fundamentally different issues require three (or more) periods. At least, it is not clear to what extent/range and under what circumstances the results derived in a two-period model could be generalized to a T-period model. As a result, the development of the T-period model has great potential to be an important tool for future research.

The Degree of Individual Heterogeneity in the Economy

Citation Information: Mishra, N.K., Ali, I., Imam, A., Khatoon, A., Olarewaju, O.M., Khan, I.A., & Baig, A. (2021). An analysis on accounting disclosure models for modeling the mathematical signals perspective. *Journal of Management Information and Decision Sciences, 24*(S6), 1-31.

This feature or attribute, of which many possibilities have been discussed in the literature, is a construct or model property that is a necessary "simplification of strategic assumptions" in the face of the complexity of reality in market settings. First of all, there is the case where all individuals are assumed to be completely homogeneous. As a practical matter, this is tantamount to assuming that there is only one individual in the economy (often referred to as the "consensus" individual). It is almost self-evident that in a pure exchange economy, there would be no information problems of interest to deal with under this characterization. Put simply, in these economies, there is no question of what the final distribution will be. If we want to overcome an economy consisting of homogeneous individuals, we have to consider whether the (ex-ante) beliefs are homogeneous or not. The same is true for utility-of-consumption-bundles. It could be argued that the assumption of uniformly homogeneous beliefs is too restrictive in the welfare analysis of information. One could argue that the uniformly homogeneous beliefs assumption is too restrictive in information welfare analysis, since many outcomes may be highly sensitive to the homogeneous beliefs assumption.

Dimensions of Commodity Space

In most general equilibrium models, there is only one consumable commodity, indexed by state and date. Analogous to this, the analysis of information in general equilibrium is limited to single-good economies. In an equilibrium model under incomplete markets, the introduction of a large number of goods (searched not only by state and date but also by type at the same time) would suggest the need to revisit the standard notion of economic efficiency. Most of the points made in "(ii) Number of consumption periods" above will apply equally here.

Endogenous Production *vs.* Exogenous Production (Pure Exchange)

Very few models examine the role of information in the production economy. Not surprisingly, the majorities of these models either impose severe constraints on the setting or use ad hoc assumptions. The basic question in an endogenous production economy is, of course, to identify under what conditions information has a positive social value. It is quite natural to be tempted to believe that the basic problems in an endogenous production economy are plainer than those in the case of a pure exchange economy, which might lead one to suggest that endogenous production models are a worthwhile object of study. However, one soon discovers that these models are extremely esoteric. This is because there are fundamental questions that are essentially unresolved, such as the well-known issues related to the theory of the firm in an incomplete market.

We are not familiar with what Ohlson & Buckman (1980) mean by "a theory of the firm in a competitive economy under incomplete markets," but we are interested in the "real effects perspective" literature in accounting, which argues that accounting information affects managers' production and investment decisions, and we have also followed part of the "feedback effects" literature in finance, which argues that stock prices both reflect and affect firms' expected cash flows. In the early stages of the analysis of information models in 1980, we were deeply moved by the suggestion of a production economy as opposed to a pure exchange economy.

Economic Regime or Structure of Transaction Coordination

A competitive market economy can be described in terms of a set of rules that determine the sequence of available securities (or corporate shares) and the beginning (and end) of general and specific markets in that economy. There are naturally many possibilities for the choice of transaction coordination, and the information issues will be dealt with in various/different regimes. First, let us consider trade coordination, where all trades are made after receiving a (random) signal

Citation Information: Mishra, N.K., Ali, I., Imam, A., Khatoon, A., Olarewaju, O.M., Khan, I.A., & Baig, A. (2021). An analysis on accounting disclosure models for modeling the mathematical signals perspective. *Journal of Management Information and Decision Sciences, 24*(S6), 1-31.

from a publicly available source. The available commodities here are current consumption claims and future consumption claims (more generally, corporate stock). I will call this regime a "no signal insurance regime" because each individual will not have the opportunity to participate in transactions that allow for the transfer of wealth between individuals with different signals. In other words, this transactional coordination is not a setup that can take advantage of transactional opportunities enabled by information structures, nor is it a setup that can coordinate allocations under different signals. As one possible improvement, let us assume that two rounds of transactions are possible under the same combination of commodities: current consumption claims and future consumption claims. The first round takes place before the information is received, and the second round takes place after the signal is received. The efficiency achieved by this regime depends on the characteristics of the price structure that is endogenously created in the second round of trading. Dare we point out that this trading structure, called a repeating market, may or may not achieve the desired level of efficiency? The problem is that commodities traded in a set of regimes do not allow for a direct transfer of wealth between individuals as a function of the signals they receive. If commodities allow for wealth transfer under different signals, that is, if commodities are indexed in the same way as firms, then one round of trading is sufficient. In short, one very important factor that determines the degree of efficiency achieved through trade coordination is the ability of trade coordination to transfer wealth under different signals and coordinate plans under different signals, and the economic regime in which commodities are indexed or labeled by potential signals is important. The regime is important, and such indexed or labeled securities become a separate commodity. Of course, this kind of argument is not new. However, the method used to analyze the impact of information on investors' decision problems differs in several ways from the method used for goods in general. The first is that the basic concern is the (relative) efficiency of a set of alternative trade arrangements and how they compare with each other. Such a comparison would require not only considering a competitive market economy, but also a "benchmark" or "planned" economy. Second, we need to consider both complete and incomplete securities markets. Finally, different information structures must be considered, and the impact of each information structure must be considered.

Heterogeneous *vs.* Homogeneous Information

These two cases are fundamentally different in terms of how they affect the analysis and the issues arising from that analysis. Homogeneous information, of course, is remarkably straightforward to handle, and standard competitive equilibrium conditions can be described without troubling problems. Such a simplification would not be applicable when heterogeneous information is at issue. The overall view of the semantic content of individual rationality must be re-examined in the setting of heterogeneous information. Similarly, there is another related issue: the problem of the existence of competitive equilibrium and the problem of an appropriate formulation of economic and information efficiency. A review of this area will reveal that the issues addressed in these two cases are significantly different. Where it may be difficult to characterize the final allocation due to the complexity of the equilibrium conditions, certain problems are imposed on the welfare analysis.

As mentioned above, Ohlson & Buckman (1980) pointed out seven factors (effectively, six factors except for (i)) that should be taken into account when constructing a useful analytical model for cutting into the accounting disclosure area. These are: (1) the number of consumption points, (2) the degree of heterogeneity of individuals in the economy, (3) the dimensions of commodity space, (4) endogenous versus exogenous production (pure exchange), (5) the structure of economic regimes or transaction arrangements, and (6) heterogeneous versus homogeneous information. It should be noted that Ohlson & Buckman (1980) discuss information analysis with an eye toward

Citation Information: Mishra, N.K., Ali, I., Imam, A., Khatoon, A., Olarewaju, O.M., Khan, I.A., & Baig, A. (2021). An analysis on accounting disclosure models for modeling the mathematical signals perspective. *Journal of Management Information and Decision Sciences, 24*(S6), 1-31.

general equilibrium analysis in its early stages (up to about 1980). Bearing in mind that Ohlson & Buckman (1980) are a discussion of information analysis with general equilibrium analysis in its early stages in mind, it is necessary to always be aware of the existence of these disclosure model elements when comparing and examining accounting disclosure models. This is because the differences in the treatment of these basic elements lead to the characterization of individual models, which may be useful in comparing models with each other.

"DISCRETIONARY-BASED DISCLOSURE STUDIES"

What Verrecchia (2001) meant by "discretion-based disclosure" was what discretion would management or the firm exercise in disclosing information? As for the specific content of discretion, will management disclose or withhold information? Under what circumstances would such disclosure or non-disclosure occur? Does management ever disclose untrue, false, or manipulated information? If so, under what circumstances, and what are management's motivations or objectives in deciding to disclose or not disclose? What are the motivations or purposes for management to make disclosure or non-disclosure decisions, and do these motivations or purposes encourage distortion or manipulation of disclosure content? If so, under what circumstances? These are just a few of the questions. The "discretion-based disclosure research" area is often modeled based on an argument called "unraveling," which is generally applied to the analysis of "voluntary disclosure" by managers or companies, as explained below. In other words, "discretion-based disclosure research" questions the disclosure incentives of managers and companies. Therefore, it can be said that this is a research area that attracts a great deal of natural interest in the accounting field, such as "what should be the disclosure strategy of companies"? In the following, I will try to be as concise as possible and trace the arguments of Verrecchia (2001) and Stocken (2012), who surveyed the same area, as Verrecchia's (2001) "discretion-based disclosure research" from their perspective.

Key Issues in "Discretionary-Based Disclosure Research"

Verrecchia (2001) points out that what discretion does management or the firm exercise in disclosing information that may be useful in evaluating the firm and of which management or the firm may know? Economists have long discussed various avenues through which the seller-specific adverse selection problem of selling an asset to a potential buyer while simultaneously withholding the quality of that sold asset can drive a seller to full disclosure to the buyer. The rationale behind this result is that a rational buyer would interpret the withholding of information as undesirable information about the value or quality of the asset. As a result, the buyer discounts the value of the asset to the point where it is in the seller's best interest to disclose the information, no matter how undesirable. The view that withholding information can be "unraveled" by rational buyer behavior led to one evolving result that forms the basis for nearly all subsequent research on this topic.

While a significant portion of financial reporting is mandatory (e.g., quarterly reports, annual reports, proxy statements, etc.), management may still have additional information that is not required to be disclosed but is nonetheless useful in assessing the prospects of the enterprise. As a result, under what conditions would management disclose or withhold such information? Early papers in the field of accounting on this question suggested that when management's objective is to maximize the firm's current market capitalization and there are costs associated with disclosure of information, there exists an equilibrium in which information that is desirable to increase the firm's current market capitalization is disclosed and information that is undesirable to increase the firm's current market capitalization is withheld. In other words, there is an equilibrium in which not all information is disclosed. In particular, it is important to note that there are situations in which

Citation Information: Mishra, N.K., Ali, I., Imam, A., Khatoon, A., Olarewaju, O.M., Khan, I.A., & Baig, A. (2021). An analysis on accounting disclosure models for modeling the mathematical signals perspective. *Journal of Management Information and Decision Sciences, 24*(S6), 1-31.

information is withheld even though investors have "reasonable expectations" about the content of the information, *i.e.*, they infer that the withheld information is not desirable. In equilibrium, there may be a variety of costs that support withholding information, but perhaps the most compelling one is the cost of disclosing proprietary information.

The features associated with these early papers have produced several compelling models of voluntary disclosure. Of these, the following three arguments are of particular interest. (A) relying on exogenous occupancy costs to explain information withholding, (B) relying on truthful reporting, and (C) relying on management's objective of boosting current market capitalization, even if boosting a firm's current market capitalization jeopardizes future firm value.

Concerning (A), the discussion includes the possibility of withholding information based on uncertainty about whether management is an information holder and uncertainty about the "type" of management or company. This is because it is thought that these also function as a kind of disclosure cost. About (B), for example, there is an argument that one of the keys to the reliability of management's disclosure is based on the persuasion game and the cheap-talk game, which loosen the constraint or assumption that management will release truthful information, keeping in mind the case of the provision of future-oriented information.

Verrecchia (2001) makes the following points regarding (C). In reviewing the accounting literature, one troubling problem that exists in the early papers is that they still rely on the assumption that management's objective in exercising discretion in disclosure is to increase the current market capitalization of the company, even if the practice of increasing the market capitalization of the company is an event that jeopardizes future returns. There are various arguments for/against management's interest in the current level of market capitalization rather than the future value of the company through accounting, including those pointing to the incompleteness of management contracts and those based on anecdotal evidence in the corporate media, such as the belief that management is conditioned to believe that it will be correctly/truly valued based on the benchmark of maximizing current market capitalization regardless of its contracts.

After summarizing the development of discretionary disclosure research based on the "unlabeled theory" (until around 2000) as described above, Verrecchia himself points out that a series of "sketch models" based on the idea of "proprietary cost" for disclosure, which he first pointed out in Verrecchia (1983), is appropriate to characterize the development of discretion-based disclosure research (Verrecchia, 2001).

However, given the problem of management's drive for market capitalization, one alternative model for motivating voluntary disclosure would be to follow the general outline of the original story, based on the concept of occupancy costs, and show how these occupancy costs arise endogenously in a duopoly game played between two companies seeking to maximize future (rather than current) market returns.

A "Sketch Model" of "Discretion-Based Disclosure Research"

Having stated the above, Verrecchia (2001) sequentially presents a "sketch model" based on four multiple occupancy models titled as follows

1. Constant occupancy costs (Model #1) Assuming that firms maximize their current value, discuss how uncertainty about the existence of constant occupancy costs or withheld information can lead to an equilibrium in which information is disclosed in some cases and withheld in others.
2. Endogenous and variable occupancy costs (Model #2) We allow for endogenous and variable occupancy costs, relax the assumption of constant costs, and continue to assume that firms seek to maximize their current value. An exciting/challenging feature of Model #2 is that it suggests that the optimal ex-post disclosure policy is one of full disclosure, while at the same time it suggests that the optimal ex-ante disclosure policy is one of non-disclosure. This illustrates the potential inefficiency of discretionary disclosure adjustments.

Citation Information: Mishra, N.K., Ali, I., Imam, A., Khatoon, A., Olarewaju, O.M., Khan, I.A., & Baig, A. (2021). An analysis on accounting disclosure models for modeling the mathematical signals perspective. *Journal of Management Information and Decision Sciences, 24*(S6), 1-31.

3. Maximization of Expected Corporate Value (Model #3) we extend the analysis to a situation of multiple occupations where firms adopt disclosure policies that maximize expected profits. In other words, companies maximize the future value, not the current value. Here, we also point out that, in contrast to disclosure adjustments that allow firms ex-post discretion in disclosure, there may be ex-ante, pre-committed adjustments that dominate such ex-post disclosure adjustments.
4. Disclosure in a "cheap-talk" setting (Model #4) Extend the duopoly setting further to a setting where there is no precondition that the company will disclose the truth.
5. Referring again to the earlier classification of persuasive models of voluntary disclosure as particularly interesting in prior papers, Model #1 is (A) a "sketch model" for papers that rely on exogenous occupancy costs to explain information withholding, Model #2 and Model #3 are said to be the most troublesome issues, (C) a "sketch model" for papers that rely on management's its core. This paper has two important features. First, involuntary disclosure decisions under a game of multiple occupancies, managers may make decisions involving future firm value, which solves the problem of the assumption that managers seek to maximize current value. Second, the multiple occupancy setting cleverly characterizes how the release of information endogenously generates occupancy costs. These are aspects of desirable characteristics. On the other hand, there are some criticisms of this approach. First, there is little additional insight relevant to revealing how the cost in question changes endogenously once the character of the cost has been identified (*i.e.,* once the fact that it is occupational has been identified). The second is that the duopoly game does not, by itself, prevent the "unlabeled" of withheld information in the absence of some additional modeling characterization. The reason for this is that if two firms are competing in the same (or similar) product market, the act of withholding information by one firm may be interpreted by the rival firm as information that is convenient for increasing output. However, once the rival's production increases beyond a certain level, the output will negatively affect the information holder's ability to generate revenue in the relevant product market, and this will drive the information-holder to make full disclosure to the rival.

Objective of boosting the current market capitalization even if it jeopardizes the future value of the company, and Model #4 is (B) a "sketch model" for papers that rely on truthful reporting.

Since Verrecchia (2001) is based on a multiple occupation model, the inverse demand function (e.g., represented as $P = \alpha + \beta\tilde{Y} - x$ in Model #1) is used. Here, P is the price, $\tilde{Y}$ is some sort of proprietary information about the next period's price known only to the firm in question, x is the current period's output produced by the firm, and α and β are certain positive constants) and the firm's profit function (e.g., represented as $Max_x xE[\tilde{P}|\tilde{Y} = Y]$ in Model #1). In some cases, a type of model based on a multiple occupation model assuming a product market is used, rather than a rational expectations type model with a so-called capital market in mind, and I will not go into the contents of those models here.

Classification in "Strategic Accounting Disclosures" by Stocken (2012)

Accounting disclosure occurs in an environment in which an entity communicates information to acting investors. One key feature of the financial reporting environment is that market participants have asymmetric information about a company. As a result, corporate management can strategically manage the communication of information. Of course, a rational investor would expect self-interested behavior from management when evaluating a company. Therefore, a company's strategy of optimally communicating information and investors' reactions to a company's disclosures need to be carefully considered. As "this monograph examines the analytical accounting disclosure literature in which companies strategically communicate information to investors" (Stocken, 2012), Stocken (2012) focuses on what Verrecchia (2001) calls "discretionary disclosure research" and presents a simplified model (called a "framework" in Stocken (2012)) that shares the same purpose as Verrecchia's (2001) "sketch model", and used it to draw the basic characteristics of each analytical model to organize the "strategic disclosure models" (up to around 2010)14. Comparing the two classification frameworks and examining the

Citation Information: Mishra, N.K., Ali, I., Imam, A., Khatoon, A., Olarewaju, O.M., Khan, I.A., & Baig, A. (2021). An analysis on accounting disclosure models for modeling the mathematical signals perspective. *Journal of Management Information and Decision Sciences, 24*(S6), 1-31.

correspondence between them seems to be useful for further understanding of "discretion-based disclosure research" or "strategic disclosure models".

Stocken (2012) classifies "strategic disclosure models" into three major categories as follows, and presents one to three simplified models called "frameworks" under each category.

(1) Persuasion game

The sender may withhold information, but the game limits the sender's report to what is true.

(i) Framework #1: Persuasion and Full Manifestation

(ii) Framework #2 Persuasion, Existence of Disclosure Costs, and Incomplete Manifestation

(iii) Framework #3 Persuasion, Uncertainty of Information (Possession), and Incomplete Manifestation

(2) Costless Signaling Games

The sender is free to publish a vague report or even a report intended to be misleading.

(iv) Framework #4 Costless Disclosure and Incomplete Manifestation

(3) Costly Signaling Games

A game in which the sender can misreport a signal at some cost.

(v) Framework #5 Costly Disclosure and Full Manifestation

(vi) Framework #6 Costly Disclosures, Uncertainty in Reporting Incentives, and Incomplete Manifestations

In addition, Stocken (2012) points out that a common feature of Frameworks #1 through #6 is that investors, as recipients of information, value corporate stocks at their expected value and that this assumption is equivalent to assuming that all investors have symmetric information and participate in a competitive market. This assumption is equivalent to assuming that all investors have symmetric information and participate in a competitive market, and thus investors are considered to be price takers.

The simplified model, which Stocken (2012) calls a framework to organize a series of "strategic disclosure models," is developed in a so-called capital market setting. Specifically, equilibrium is depicted as the relationship between the expected utility (more precisely, the expected payoff) of managers who disclose information on the one hand and the pricing of firms in capital markets based on the disclosed information on the other. For example, Stocken (2012, p.10) explains Framework #1 (the basic framework of the persuasion game) as follows. Let the value of the firm be $\tilde{v}$, and assume that management and investors believe that the value of the firm is uniformly distributed in the interval [0,1]. Management privately observes the actual value v of the firm and then chooses to send a single, costless message m about this value to investors. All disclosures must be truthful (*i.e.*,=v), but management can withhold information about itself (*i.e.*, $m = \phi$). The investor then takes action and evaluates the company at the expected value $P = E[\tilde{v}|\Omega]$. Where Ω denotes the investor's information set that depends on management's report or its absence. Management's expected payoff is a strictly increasing function of the investor's valuation and is given by

$$E[U(m, \tilde{v}, b)] = E[bP(m)]$$

Where b>0 is one parameter that reflects the extent to which managerial payoffs are an increasing function of investor behavior. All aspects of the game are assumed to be common knowledge, except for management's private information. It is clear from the above that the left-hand side of the above equation represents the expected utility of managers, and the right-hand side represents the pricing of firms (by investors) in the capital market.

Citation Information: Mishra, N.K., Ali, I., Imam, A., Khatoon, A., Olarewaju, O.M., Khan, I.A., & Baig, A. (2021). An analysis on accounting disclosure models for modeling the mathematical signals perspective. *Journal of Management Information and Decision Sciences, 24*(S6), 1-31.

Contrast between the framework of "Strategic Accounting Disclosure" and the sketch model of "Discretionary-Based Disclosure Research"

Bearing in mind that the base of the framework referred to by Stocken (2012) and the "sketch model" of Verrecchia (2001) are different (the former is a rational expectations type model based on the so-called capital market, while the latter is a multiple occupation model), let us turn our attention to the correspondence between them. Aside from the fact that Framework #1, which was

partly pointed out above, depicts the prototype of the "unlabeled theory," Frameworks #2 and #3 correspond to Verrecchia's (2001) Model #1 (a constant occupation cost model that includes the uncertainty of information existence type model). Also, framework #4 is based on the so-called "cheap-talk game" and can be understood as corresponding to Verrecchia's (2001) model #4, at least in this sense. What about Framework #5 and Framework #6 under the costly signaling game? Can we find some kind of commonality between Verrecchia's (2001) Model #2 and Model #3?

Stocken's (2013) explanation of costly signaling games is as follows. Here, the costly signaling game discusses a framework in which management incurs direct costs from publishing manipulated reports, management's manipulation of reports is not observed by investors, and the interests of management and investors are not aligned, bearing in mind the situation often found in financial reporting environments where management may manipulate information despite certain management costs. Such a model, characterized by the fact that investors do not observe management's operations and management incurs costs in manipulating reports, is often called a signal-jamming model. Because signal-jamming models are one particularly descriptive way of describing the financial reporting environment when management can manipulate reporting at some cost, these frameworks have been increasingly used in the accounting literature to model mandatory reporting and, more recently, to study voluntary disclosure behavior. In this signal-jamming model, in the absence of uncertainty about management's reporting incentives, there is an equilibrium in which management manipulates reporting and investors fully anticipate this manipulation and discount management's reporting appropriately. In other words, Framework #5 deals with a signal-jamming model in which investors are uncertain about management's private information, but management's reporting incentives are common knowledge. In Framework #5, management's reporting incentives are common knowledge, which means that while management manipulates the reporting of its private information, investors reasonably anticipate this manipulation and filter out management's private information from management's reporting altogether, and as a result, management's manipulation of this reporting does not affect the informativeness of this reporting. In contrast, Framework #6 relaxes the assumption that management's reporting incentives are common knowledge and instead assumes that only management privately observes its reporting incentives (*i.e.*, investors do not observe management's reporting incentives). As a result, investors will not be able to completely filter out management's reporting bias, while management will not always be harmed by the possibility of manipulating its reporting, even if the manipulation reduces the informativeness of the reporting. Framework #6 deals with a signal-jamming model that makes a new assumption about the uncertainty of management's reporting incentives.

Verrecchia's (2001) Model #2 and Model #3 in the "Discretionary-Based Disclosure Study" are built around the concept of occupancy cost, based on the question of why managers try to boost current market capitalization through disclosure (Category (C)), even if boosting current market capitalization would jeopardize future corporate value. This awareness of the problem can be seen as dealing with a situation in which investors (capital markets) do not place confidence in the information disclosed, and even if there is a possibility of lowering the company's valuation, that is, even if there is a possibility of incurring costs, the management side adds discretion to the information disclosed or distorts the information disclosed to boost the company's valuation. In this

Citation Information: Mishra, N.K., Ali, I., Imam, A., Khatoon, A., Olarewaju, O.M., Khan, I.A., & Baig, A. (2021). An analysis on accounting disclosure models for modeling the mathematical signals perspective. *Journal of Management Information and Decision Sciences, 24*(S6), 1-31.

sense, Verrecchia's (2001) Model #2 and Model #3 are the multiple occupancy model versions of the signal-jamming model. Thus, in Model #2, through the Cournot duopoly model, in which there are two firms, one with proprietary information and the other without, the discretion of the firm with proprietary information to disclose or not to disclose is questioned in response to changes in the cost of proprietary information, which is regarded as the cost of reporting manipulation/disclosure discretion. In Model #3, through the Cournot duopoly model where there are two companies with their proprietary information, we can understand more clearly that the discretion of disclosure or nondisclosure is being questioned when management aims to maximize future value without being concerned about increasing (current) corporate value.

As Stocken (2012) points out, the signal-jamming model, which deals with situations in which management may exercise discretion over what information to disclose at some cost, is an analytical model that is equally well suited to mandatory disclosure, which is what we are interested in, in addition to voluntary disclosure. If Verrecchia's (2001) Model #2 and Model #3 depict discretionary disclosure behavior of the type of disclosure/non-disclosure (including disclosure under certain circumstances (partial disclosure)) using a multiple occupancy model, limiting the scope to voluntary disclosure, with a common awareness of the problem with the signal-jamming model. In contrast, Stocken's (2012) Framework #5 and Framework #6 use the so-called rational expectations type model, which assumes a general capital market in the signal-jamming model, to depict mandatory disclosure behavior such as reporting manipulation within the scope allowed by Generally Accepted Accounting Principles (GAAP).

Basic Types of Signal-Jamming Models

From the perspective of "truthful reporting," a situation in which management or the company has discretion over the information it discloses, which is distinguishable from a so-called "cheap-talk" setting in which there are no restrictions on misreporting, is a natural reporting environment, even if we keep in mind mandatory disclosure and verifiable accounting information. Typically, there is a choice between several accepted accounting and disclosure procedures applied to the same accounting event. There can also be unintentional accounting measurement errors. In this sense, the signal-jamming model is considered to be one of the most useful basic models for analyzing accounting disclosures. The following section traces framework #5 of Stocken (2012) to understand the framework of the signal-jamming model.

In the signal-jamming model, if there is no uncertainty about the reporting incentives of management, the sender of the information or message (Framework #6 depicts a situation where this reporting incentive uncertainty exists), then there is an equilibrium in which management manipulates reporting and the recipient, the investor, fully anticipates this manipulation and appropriately discounts the sender's reporting. To illustrate this result with a mathematical expression, consider the following signal-jamming model in which investors are uncertain about management's private information, but management's reporting incentives are common knowledge. The value of the firm is a single random variable, denoted by $\tilde{v}$. The prior beliefs of the players (managers and investors) are that the value of the firm has unbounded support with finite mean μ_v.

Management privately observes the actual value v of the company. Management reports message $m \in \mathcal{M}$. The investor takes the action represented by P, which evaluates the company by its expected value, given the arbitrary information contained in management's report m. In other words, it is $P(m) = E[\tilde{v}|m]$. Management's expected payoff is given by

$$E[U(m, \tilde{v}, b)] = E\left[bP(m) - \frac{1}{2}(m - \tilde{v})^2\right]$$

where b>0 is a parameter that reflects the degree to which the players' interests are in disagreement. The players' interests are not in agreement. This is because management's payoff is

Citation Information: Mishra, N.K., Ali, I., Imam, A., Khatoon, A., Olarewaju, O.M., Khan, I.A., & Baig, A. (2021). An analysis on accounting disclosure models for modeling the mathematical signals perspective. *Journal of Management Information and Decision Sciences, 24*(S6), 1-31.

an increasing function of investor behavior P(m), regardless of the actual state v, but management bears the direct cost associated with publishing a report m that differs from its private information v, and this cost is independent of the investor behavior induced by the message. All aspects of the game are common knowledge, except for management's private information and the extent/degree to which management manipulates the report.

In all equilibria, the following conditions must be met

(i) Management maximizes its payoffs, given the presumed behavior of investors. That is, for all v, m(v) maximizes $b\hat{P}(m) - \frac{1}{2}(m-v)^2$. Here (hat) represents the player's guess.

(ii) Investors evaluate companies based on their expectations, given management's message. In other words, for all m, $P(m) = E[\tilde{v}|m; \hat{m}(v)]$

(iii) Investors' inferences about management's reporting strategy must be self-fulfilling, and at the same time, management's inferences about investors' strategy must be self-fulfilling. That is, $\hat{m}(v) = m(v)$ and $\hat{P}(m) = P(m)$.

In the following, we focus on linear equilibrium, which is common in the accounting literature.

Proposition 1: There exists one unique linear equilibrium in which management reports $m(v) = v + b$ and investors value the firm at $P(m) = E[\tilde{v}|m] = m - b$. Management's ex ante expected utility is equal to

$$E[U(m, \tilde{v}, b)] = b\mu_v - \frac{b^2}{2}$$

Proof. Let $\hat{P} = \hat{\beta}_0 + \hat{\beta}_m m$ represent management's conjecture about the investor's reaction function, and let $\hat{m}(v) = \hat{\alpha}_0 + \hat{\alpha}_v v$ represent investor's conjecture about management's reporting strategy.

In the conjectured equilibrium, management chooses m, the solution to the equation.

$$max_{m \in \mathcal{R}}\left[b\hat{P}(m) - \frac{1}{2}(m-v)^2\right] = max_{m \in \mathcal{R}}\left[b\left(\hat{\beta}_0 + \hat{\beta}_m m\right) - \frac{1}{2}(m-v)^2\right]$$

The above equation is obtained by substituting the investor's reaction function for the manager's guess. Differentiating the manager's objective function concerning m and using the first-order condition, we obtain $m(v) = b\hat{\beta}_m + v$, and the second-order condition is satisfied because the manager's objective function is concave. The manager's equilibrium strategy is given by $m(v) = \alpha_0 + \alpha_v v$. Here it is $\alpha_0 = b\hat{\beta}_m$and $\alpha_v = 1$. Since management's reporting function is invertible, investors can infer v from management's reports. Thus, the expected value of firm value is $P(m) = \beta_0 + \beta_m m$, where $\beta_0 = -b\hat{\beta}_m$and $\beta_m = 1$. In equilibrium, the players' guesses must be self-fulfilling, so we end up with the results $\alpha_0 = b$, $\alpha_v = 1$, $\beta_0 = -b$, and $\beta_m = 1$.

Finally, management's prior expected utility is given by the following equation.

$$\begin{aligned} E[U(m, \tilde{v}, b)] &= E[bP(m) - \tfrac{1}{2}(m(\hat{v}) - \tilde{v})^2] \\ &= E[b\left(-b + (b + \tilde{v})\right) - \tfrac{1}{2}(b + \tilde{v} - \tilde{v})^2] \\ &= b\mu_v - \frac{b^2}{2} \end{aligned}$$

The above equation can be obtained by substituting $\alpha_0 + \alpha_v v = b + v$ for m(v) and $\beta_0 + \beta_m m = -b + m = -b + b + v$ for $P(m)$.

One key feature of this costly reporting equilibrium is that in equilibrium, management manipulates its reporting, even if investors fully anticipate the level of bias and discount management's reporting appropriately. Thus, investors value firms at their actual value. In other words

$$P(m) - v = (m(v) - b) - v = 0$$

and

$$Var(\tilde{v}|m) = 0$$

Citation Information: Mishra, N.K., Ali, I., Imam, A., Khatoon, A., Olarewaju, O.M., Khan, I.A., & Baig, A. (2021). An analysis on accounting disclosure models for modeling the mathematical signals perspective. *Journal of Management Information and Decision Sciences, 24*(S6), 1-31.

The intuition for this result is that, given the investor's knowledge of management's objective function, the investor expects that management will skew its reporting upward. Since the investor cannot observe the actual bias introduced by management, only the bias parameter, which is always discounted in management's reports, is used. Therefore, this dissembling does not mislead investors and may be no different from management's reporting of signals that they have observed in private. However, this misrepresentation imposes reporting costs on management. One consequence of falling into the trap of this socially inefficient equilibrium is that managers are willing to commit to not disclosing signals they observe privately, given that in equilibrium they bear the cost of reporting b^2/2 and receive no benefit. If management could still commit to remaining silent, it would have enjoyed an expected payoff of bμ_v.

The above is the discussion of Framework #5 in Stocken (2012, pp. 54-57). Since Framework #5 is a sketch of the basic type of signal-jamming model, it is assumed that investors, as recipients, fully anticipate indications that have been added or distorted by the sender's management discretion. In reality, the investor's perfect expectation is not possible. As a result, management bears all the costs and suffers none of the damages, even if it adds discretion to the disclosed information. There are many possible reasons for the creation of such a situation. For example, Stocken's (2012) Framework #6 loosens the assumption that investors know management's reporting incentives and reveals that management may prefer discretionary or distorted presentation under the assumption that management's reporting motives are uncertain to investors (Framework #6 is, in this sense, more adapted to the real financial reporting environment. However, at the same time, to understand the framework of the signal-jamming model, while it is an extension of Framework #5, the operations themselves are quite complex and will not be discussed in this paper). Therefore, to develop a signal-jamming model that better reflects reality, it is extremely important to identify the facts and circumstances in which discretion may be applied and manipulated realistically. Moreover, it is equally important to determine what is considered a discretionary or manipulated indication. If managerial accounting discretion or manipulation is broadly interpreted as the degree of freedom in the content of disclosed accounting information, then there is a possibility that everything from the asset-liability approach versus the cost-earnings approach to the choice of conflicting accounting and disclosure procedures at the level of individual accounting standards can be treated as discretion, manipulation, or choice. As Stocken (2012) points out, the signal-jamming model is certainly useful and attractive as an analytical model for financial accounting under the assumption of mandatory disclosure.

CONCLUSION

The current financial accounting system is based on the premise that two economic entities play the role of making decisions on corporate production and investment on the one hand and evaluating and pricing corporate activities in the capital markets on the other, as seen in the "information provision function. As a result, the question arises as to what kind of mandatory disclosure of corporate information should be useful for investors' decision-making. Leaving aside the question of how conflicting factors such as balance sheet (stock) or income statement (flow), market value or cost, profit or cash flow, etc., are given preference and what the basis for such preference is (the so-called "evaluation criteria" for accounting information), it is safe to say that the current financial accounting system has been constructed in the manner described above, based on the statement that information asymmetries between companies and investors, as well as information asymmetries between investors, should be eliminated or mitigated. If so, financial accounting-related analytical models should clarify to what extent and in what form accounting disclosures should be incorporated into prices (stock prices) that are considered to be formed as a result of cumulative or aggregated investor behavior in the capital markets to which accounting

Citation Information: Mishra, N.K., Ali, I., Imam, A., Khatoon, A., Olarewaju, O.M., Khan, I.A., & Baig, A. (2021). An analysis on accounting disclosure models for modeling the mathematical signals perspective. *Journal of Management Information and Decision Sciences, 24*(S6), 1-31.

information is disclosed. In this sense, financial accounting-related analytical models should not be based on a grossly simplified model of the capital market process that forms the prices at which information is aggregated, but on a model that endogenizes the capital market process, specifically, a model (or at least the basic type of model) that can generally be called a "noisy rational expectations-type model," as discussed in Section 2 ("Relevance-Based Disclosure") of this paper.

The "feedback effect" literature in finance and the "real effects perspective" literature in accounting argue against the natural view of one-way information transmission from firms to capital markets, which is also the basis of the current financial accounting system design. In the "feedback effect" literature, the price (stock price) formed in the capital market reflects the expected cash flow of the company and in turn, affects the expected cash flow of the company. In the "real effects perspective" literature, it is argued that accounting information affects the production and investment decisions of firms (through the valuation and pricing of firms formed in the capital market) and that it is the transmission of information from the capital market to firms that is important and must be analyzed, contrary to the direction of information transmission that has been naturally assumed. One interesting aspect of these discussions is that in economics, where information is the main subject of study, there has been a shift in interest from informational efficiency (also called price efficiency, of course, since prices are generally the only numerical signals in mind) to the economic efficiency of numerical signals (whether the numerical signals formed lead to an efficient allocation of resources). In the field of information efficiency, the focus has shifted from price efficiency to the economic efficiency (also called real efficiency) of numerical signals, which is whether or not they lead to an efficient allocation of resources. This is because, in the traditional discussion, which is concerned only with the informational efficiency of prices as a numerical signal, it has been assumed that informationally efficient prices naturally lead to efficient resource allocation.

If, as the "real effects perspective" literature argues, the way accounting measurement and disclosure are carried out influences companies' production and investment decisions, in other words, if accounting information has real effects, what the content/content of accounting disclosure is, including whether or not discretion can be added, suddenly becomes an important issue. This is because the content of accounting disclosure not only affects the informational efficiency of the numerical signal of price (stock price) but also, in turn, the economic efficiency of the company's production and investment. One aspect of the situation, such as what kind of information content managers use in accounting disclosures, can be captured by the signal- jamming model discussed in Section 3 ("Discretionary-Based Disclosure"). Recently, the "real effects perspective" is also called the Kanodia/Stein approach after Stein, who is considered to be one of the developers of the signal-jamming model. Financial accounting-related analytical models must be based on a model called the signal-jamming model (or at least its basic form).

Based on the idea of "transaction cost economics," which positions markets and organizations as alternative resource allocation mechanisms, we view the accounting system as an artificial signal-forming system to cope with the incompleteness or imperfection of markets. The accounting information (especially the accounting profit) produced by the accounting system complements and substitutes for the market price, and thus acts as a numerical signal to coordinate economic transactions or to allocate resources among firms. In our view of accounting functions, which we call the "number signal perspective," An analytical model that balances and combines the basic features of a "noisy rational expectations type model" that endogenizes capital market processes with the basic features of a signal-jamming model that can handle disclosure content with a degree of freedom is desirable. More specifically, a "noisy rational expectations type model" that maintains a certain degree of discretion and concreteness in accounting disclosures, or a signal-jamming model that allows for a certain endogenization of capital market processes, keeping in mind the characteristics of accounting information (especially accounting profit) that acts as a

Citation Information: Mishra, N.K., Ali, I., Imam, A., Khatoon, A., Olarewaju, O.M., Khan, I.A., & Baig, A. (2021). An analysis on accounting disclosure models for modeling the mathematical signals perspective. *Journal of Management Information and Decision Sciences, 24*(S6), 1-31.

numerical signal, is the desirable analytical models necessary to analyze the function of accounting information (accounting profit) as we envision it. This is because the function of accounting information (accounting profit) as a numerical signal can be depicted in opposition to price as a numerical signal, and the two numerical signals, their specific functions, and the interaction between them can be explicitly analyzed.

REFERENCES

Admati, A.R., & Pfleidere, P. (1986). A monopolistic market for information. *Journal of Economic Theory, 39,* 400−438.

Admati, A.R., & Pfleidere, P. (1988). Selling and trading on information in financial markets. *Review of Financial Studies, 78,* 96−106.

Ahmad, S.R., Senan, N.A.M., Ali, I., Ali, K., Khan, I.A., & Baig, A. (2021). Investor reaction to the discovery of accounting fraud: the period from the discovery of the fraud to the completion of the correction. *Academic Journal of Interdisciplinary Studies, 10*(6), 171-190.

Ahmad, S.R., Olarewaju, O.M., Ali, I., Baig, A., & Khan, I.A. (2021). Impact of accounting fraud on stock price formation before its discovery-the period from the start of fraud to its discovery. *Academy of Entrepreneurship Journal, 27*(5S), 1-19.

Ahmad, S.R., Olarewaju, O.M., Ali, I., Baig, A., & Khan, I.A. (2022). An analysis of Malaysian SMEs' access to public financial assistance. *Universal Journal of Accounting and Finance, 10*(1).

Bond, P., Edomans, A., & Golstein, I. (2012). The real effects of financial markets. *Annual Review of Financial Economics, 4,* 339−360.

Darrough, M.H. (1993). Disclosure policy and competition: Cournot *vs.* Bertrand. *Accounting Review, 68,* 534−561.

Diamond, D.W., & Verrecchia, R.E. (1981). Information aggregation in a noisy rational expectations economy. *Journal of Financial Economics, 9,* 221−235.

Fisher,P.E., & Vrrecchia, R.E. (2000). Reporting bias. *Accounting Review, 75,* 229−245.

Goldstein, I., & Yang, L. (2017). Information disclosure in financial markets. *Annual Review of Financial Economics, 9,* 101−125.

Goldstein, I., & Yang, L. (2019). Good disclosure, bad disclosure. *Journal of Financial Economics, 131,* 118−138.

Grossman, S.J. (1976). On the efficiency of competitive stock markets where trades have diverse information. *Journal of Finance, 31*, 573−585.

Grossman, S.J. (1981). The role of warranties and private disclosure about product quality. *Journal of Laws and Economics, 24,* 461−483.

Grossman, S.J., & Hart, O.D. (1981). Disclosure laws and takeover bids. *Journal of Finance, 35,* 323−334.

Kanodia, C. (2007). Accounting disclosure and real effects. *Foundation and Trends in Accounting, 1*(3), 1-95.

Kanodia, C., & Sapra, H. (2016). A real effects perspective to accounting measurement and disclosure: Implications and insights for future research. *Journal of Accounting Research, 54,* 623−676.

Khan, I.A. (2018). The effect of rating changes on stock returns: An empirical investigation. *International Journal of Research in Management, Economics and Commerce, 8*(3), 9–14.

Khan, I.A. (2011). Kaizen: The Japanese strategy for continuous improvement. *International Journal of Business and Management Research, 1*(3), 177–184.

Khan, I.A. (2012). An overview of insider trading: US *v/s.* India. *Journal of IMS Group, Listed in Cabell's Management Directory, 9*(1), 59–62.

Khan, I.A. (2010). Forensic accounting: A new paradigm for niche consulting. *Proceedings of the National Conference on Contemporary Issues and Emerging Trends in Management and Technology.*

Khan, I.A. (2018). Empirical evidence on dividends as a signal of firm value. *VSRD International Journal of Business and Management Research, 8*(1), 23–30.

Khan, I.A. (2021). Time-driven activity costing and its application in public hospitals. *Management Accountant, 56*(04), 89–93.

Kyle, A.S. (1985). *Continuous auctions and insider trading.* Econometrica, 53, 1315−1335.

Ohlson, J.A., & Buckman, A.G. (1980). Toward a theory of financial accounting. *Journal of Accounting Research 18,* 537−551.

Stein, J.C. (1989). Efficient capital markets, inefficient firms: A model of myopic corporate behavior. *The Quarterly Journal of Economics, 104,* 655−669.

Stocken, P.C. (2012). Strategic accounting disclosure. *Foundation and Trends in Accounting, 7,* 1−100.

Verrecchia, R.E. (1982). The use of mathematical models in financial accounting. *Journal of Accounting Research 20,* 1−55.

Citation Information: Mishra, N.K., Ali, I., Imam, A., Khatoon, A., Olarewaju, O.M., Khan, I.A., & Baig, A. (2021). An analysis on accounting disclosure models for modeling the mathematical signals perspective. *Journal of Management Information and Decision Sciences, 24*(S6), 1-31.

Verrecchia, R.E. (1983). Discretionary disclosure. *Journal of Accounting and Economics, 5,* 365−380.
Verrecchia, R.E. (2001). Essays on disclosure. *Journal of Accounting and Economics, 32,* 97−180.

Citation Information: Mishra, N.K., Ali, I., Imam, A., Khatoon, A., Olarewaju, O.M., Khan, I.A., & Baig, A. (2021). An analysis on accounting disclosure models for modeling the mathematical signals perspective. *Journal of Management Information and Decision Sciences, 24*(S6), 1-31.