A COMPARATIVE ANALYSIS OF ASSET PRICING MODELS EXPLORING FIT WITHIN INDIAN MUTUAL FUNDS

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ABSTRACT

In order to ascertain which asset pricing model best suits the risk and return structure of Indian open-ended equities mutual funds, the study evaluates three alternative models: CAPM, APT, and Fama-French. Utilizing a five-year dataset spanning from March 2018 to March 2023, the study looks at the performance of 20 Indian funds. Principal Component Analysis and regression analysis are used to assess the models' explanatory power. For data processing and visualization, the study makes use of Jamovi statistical software and Microsoft Excel. The research findings have the potential to improve the investing and risk-return decision-making of Indian mutual fund managers by providing them with a better understanding of the drivers of risk and return.

Keywords: Capm, Fama French Models, Asset Pricing Models.

INTRODUCTION

Economic and policy shifts affect future company cash flows, which in turn impact investor anticipations for return and risk tolerance. Liberalization has altered the Indian economy, impacting the capital market and requiring a scientific financial approach. The emergence of mutual funds in India has transformed financial behaviour's by providing a means to achieve objectives through asset distribution. It is essential to comprehend the connection between risk and return, even when utilizing diversification. Asset pricing models aid investors in assessing anticipated returns by considering risk, directing choices in a convoluted financial environment. The central focus is on the effectiveness of these models in capturing distinct risk-return patterns in Indian mutual funds. This research examines the CAPM, APT, and the Fama-French Four-Factor Model to gain insight into the factors impacting returns in the Indian equity mutual fund market. The study seeks to identify the most effective model for describing the riskreturn dynamic in Indian mutual funds through a comparison of their explanatory abilities and adjusted R-squared values. The main objective is to offer investors and fund managers valuable information to improve investment decision-making, portfolio construction, performance evaluation, and risk management in the Indian equity mutual fund market.

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THEORETICAL BACKGROUND

Capital Asset Pricing Model

In modern finance theory, the Capital Asset Pricing Model (CAPM) was developed during the 1960s. William Sharpe is frequently given recognition for its creation, although Jack Treynor, Jan Mossin, and John Lintner also played important roles. Following Harry Markowitz's 1952 Mean-Variance Portfolio Theory (MPT), the CAPM model forecasts asset returns by considering their inherent risk. It differentiates between systematic (non-diversifiable) and unsystematic (specific) risks, highlighting the significance of taking both into account when planning investments. The CAPM model describes market risk as a non-diversifiable risk that impacts returns on securities. It illustrates the influence of risk-free rate and risk premium on returns, with the risk premium determined by beta sensitivity. The model's straight correlation between expected return and risk enables a balance between them, helping in making investment choices.

The Camp Formula

The expected return (ER) of an asset is used to determine using the CAPM formula, which takes into

account its systematic risk i.e. (beta, β).

 $ER=Rf+\beta *(Rm-Rf)$

Where:

Rf: indicates the risk-free rate, or the yield on a low-risk investment such as government bonds.

B: indicates Beta, the volatility of an asset in relation to the market as a whole

Rm: indicates expected market return

(Rm – Rf): indicates Market risk premium, or the anticipated excess return of the market over the risk-free rate.

Relative to the overall market, the beta coefficient (β) represents the volatility of an investment in the market. In the Capital Asset Pricing Model (CAPM), it is a crucial element. Beta can be calculated using the following formula:

Beta (β) = Covariance (both Return on Investment and Return on Market) / Variance (Market Return).

The Capital Asset Pricing Model (CAPM) assumes that investors prefer bigger rewards over greater risk due to risk aversion. It assumes that investors maintain a well-diversified portfolio on the efficient frontier, which yields the best expected return for a given level of risk. The model assumes that every investor anticipates the same risks and returns in the future and that taxes or transaction fees are not present. The CAPM Model has been thoroughly studied and discussed in the finance sector ever since it was first introduced. While there is evidence to back its credibility in established markets, critiques stem from impractical assumptions such as flawless market efficiency. Research has found that CAPM has difficulty explaining factors such as size and value effects. Other models like APT and Fama-French Four-Factor Model have been created to overcome the shortcomings of CAPM for a more precise examination of the correlation between risk and return, particularly in developing nations such as India.

Apt (Arbitrage Pricing Theory) Model

The APT Model, created by Stephen A. Ross in 1976, suggests that a security's return is connected to systematic factors and the risk-free rate. APT discovers the root reasons behind uncertainty that impact security risk. Variables such as inflation, interest rates, and economic growth influence the returns of assets as well as market risk. APT suggests that there is a direct correlation between asset returns and macroeconomic variables. Market inefficiencies present chances for earning money by taking advantage of incorrectly priced assets, which arbitrageurs use to move prices closer to balance. APT also presupposes rational investors who want to maximize returns according to their risk tolerance, without considering taxes or transaction costs.

Apt Structure

 $ERi = \beta i 1F1 + \beta i 2F2 + ... + \beta i nFn$

Where:

ERi = Expected return on asset i β i1 to β in = Factor loadings of asset i on each risk factor (F1 to Fn) F1 to Fn = Macroeconomic risk factors influencing asset returns

APT is a risk-based portfolio management model that focuses on the relationship between factor exposure and predicted return. It eliminates the risk-free rate (Rf) and emphasizes risk premiums, which are the excess return over the risk-free rate, based on an asset's exposure to different risk variables. APT functions as an equilibrium model in markets where arbitrage opportunities are removed, suggesting that minimally risky risk-free assets provide the anticipated return for taking no risk. The factor loadings (β i) handle systematic risk, which cannot be mitigated by diversification.

Fama-French Four-Factor Model

Eugene Fama and Kenneth French discovered that β alone couldn't explain portfolio returns, leading them to create the influential Fama-French Three-Factor Model in 1993. This groundbreaking asset pricing model includes factors for market, company size, and book-to-market ratio. It addressed the limitations of the CAPM and disrupted the financial industry. The CAPM's effectiveness in linking risk and return is questioned by the Fama-French model's size and value components, profoundly altering asset valuation practices. CAPM solely considers market beta when predicting returns, failing to account for reasons behind the outperformance of specific assets. It takes into account market efficiency; however, returns can be influenced by actual inefficiencies in the world. The Three-Factor Model considers size and value factors but faces challenges with the momentum anomaly, where strong stocks maintain their outperformance and weak stocks lag behind.

Presenting the Momentum Factor (WML)

Fama and French incorporated the momentum factor (WML) into their four-factor model to address the issue. This factor focuses on the difference in returns between portfolios of stocks with strong recent performance (winners) and those with negative returns (losers). The momentum factor measures the tendency for well-performing stocks to continue outperforming, while poorly-performing stocks tend to worsen over a specific period. Adding momentum may slightly improve the model's ability to explain stock return variability.

Fama French Four-Factor Model framework

Although it is not the most popular version, the Fama-French Four-Factor Model expands on the Three-Factor Model by including Momentum (WML). Below is a breakdown of the factors and the formula:

$ERi = Rf + \beta iMkt-RF (Rm - Rf) + \beta iSMB (SMB) + \beta iHML (HML) + \beta iWML (WML)$

Where:

ERi: Expected return on asset i
Rf: Risk-free rate
βiMkt-RF: Market beta of asset i (exposure to market risk)
Rm - Rf: Market risk premium (expected excess return of the market over the risk-free rate)
βiSMB: Size factor beta of asset i (exposure to size factor)
SMB: Size factor premium (difference in returns between small and large caps)
βiHML: Value factor beta of asset i (exposure to value factor)
HML: Value factor premium (difference in returns between value and growth stocks)
βiWML: Momentum factor beta of asset i (exposure to momentum factor)
WML: Momentum factor premium (difference in returns between recent winners and losers)

LITERATURE REVIEW

Several studies by (Atodaria, et. al., 2021); (Bahl, 2006); (Gurunathan, et. al., 2021); Patel, 2016), & (Dash & Sumanjeet, 2008) appear to concentrate on the prevalence of the Fama-French model and how it can be applied in the Indian setting. This indicates a possible shift towards utilizing the multi-factor model instead of CAPM for analysing asset pricing in India.

The significance and drawbacks of CAPM are being investigated by a group of researchers (Ansari, 2000); (Hawaldar, 2011); (Diwani, 2010); (Wakyiku, 2010); (Allen & Bujang, 2009). (Allen & Bujang, 2009) in different markets like India, Uganda, and UK.

Comparison of Models: Studies conducted by (Dhankar & Singh, 2008); (Kisman & Restiyanita, 2015); & (Maxim, 2015) investigate the efficacy of CAPM and APT, potentially evaluating their performance in various markets such as India, Indonesia, and Romania.

Concentrate on Target Markets: Research conducted by (Kianpoor & Dehghani, 2016); (Nikolaos, 2009); (Diwani, 2010) & (Mudthoulad, 2023) examines the relevance of asset pricing models in particular markets or industries.

Other possible reasons: (Das & Pattanayak, 2013); (Bathia et. al., 2024) delve into key factors affecting the Indian market, proposing constraints of conventional asset pricing models and the importance of including additional explanatory variables. This analysis shows the wide range of studies on asset pricing models in different markets and situations. It also highlights the continuous investigation of different models (such as Fama-French) in addition to CAPM and the quest for a more thorough comprehension of the factors influencing risk and return.

RESEARCH METHODOLOGY

To what extent the risk-return relationship in Indian mutual funds is effectively represented by classic asset pricing models is the purpose of this research. Enhancing knowledge, guiding investment choices, improving the model, and offering important lessons for other developing economies are the objectives of this research. Three asset pricing models—CPM, APT, and the Fama-French Three-Factor Model—will be compared in this study to see if the performance differs depending on the market

capitalization of large-, mid-, and small-cap mutual funds in India. As part of the study, the best model for representing the dynamics of risk and return in the Indian context will be identified, and it will also be determined whether any new or modified models are needed to better reflect the unique characteristics of the Indian market.

With a focus on open-ended equity mutual funds, a significant category in the Indian market, this study seeks to close gaps in asset pricing models in the context of India. While earlier research may have examined a greater range of investment vehicles, this study provides a more thorough analysis that is applicable to a larger group of investors. To guarantee that the analysis appropriately depicts the condition of the Indian market, the study makes use of latest data. It also examines whether other models, such as APT, or model refinement may be necessary for the Indian setting. Additionally, the study seeks to close gaps in earlier studies that focused exclusively on the early or pre-liberalization periods, which would not accurately reflect the dynamic risk situation in the Indian market. (Exploratory research has been used in this work. Informal in nature, exploratory research relies on secondary data.)

Hypothesis of The Study

 H_1 : The risk-return connection inside Indian open-ended equities mutual funds will be better explained by the Fama-French Three-Factor Model than by CAPM and APT.

 H_2 : It's possible that several important variables affecting returns in Indian open-ended equities mutual funds are not adequately captured by the asset pricing models currently in use (CAPM, APT, Fama-French).

Sample Selection and Data Collection

A study was conducted on the top 20 equity open-ended mutual fund schemes in India, covering small, mid, and large cap categories. Data analysis was based on a 5-year time frame from March 2018 to March 2023. The list of top-performing schemes was sourced from ICRA's website, resulting in 6 small cap, 6 midcap, and 8 large cap fund schemes selected for analysis. Monthly adjusted close prices of these schemes were downloaded from Yahoo Finance for further examination of asset pricing models.

Tools for data collection: https://www.icraanalytics.com/Home/MFI

Data Required For

Capm Model Analysis

- 1. Monthly Returns of the mutual fund Schemes
- 2. Monthly Benchmark Returns (Benchmark taken into consideration for the analysis was Nifty 500 as it index represents a broad spectrum of the Indian stock market, encompassing large, mid, and small-cap companies. This makes it a suitable benchmark for comparison with open-ended equity mutual funds, which can invest across these market capitalizations.)
- 3. Monthly Risk-free rate
- 4.

Tools for data collection: Yahoo Finance (https://finance.yahoo.com/)

Apt (Arbitrage Pricing Theory) Model Analysis

The challenge in APT lies in selecting factors without guidance, leaving researchers to determine variables themselves. Despite the lack of theoretical direction, this approach has its pros and cons. Factors must be unpredictable and have a market-wide impact to be considered. Past values or external information that can easily predict returns are already reflected in stock prices and do not need to be

studied. Factors should not be specific to a single company but have a widespread impact. In selecting variables, a literature review has been conducted, focusing on macroeconomic factors such as inflation, GDP growth, interest rates, and unemployment. Monthly data for these factors will be collected from March 2018 to March 2023 to analyse their impact on returns. This method aids in realistic result interpretation and broad market impact assessment.

Gross Domestic Product (Gdp)

GDP reflects the value of goods and services produced in a country yearly, showing economic health. In an APT model, increasing GDP growth could lead to higher equity demand and expected returns. Monthly GDP data was obtained by interpolating RBI's quarterly data.

CPI (Consumer Price Index)

CPI tracks price changes for consumer goods and services, crucial for measuring inflation. An increase could mean higher costs for businesses, impacting profits. Data sourced from RBI's Indian Economy Database.

WPI (Wholesale Price Index)

WPI tracks wholesale price changes like CPI for consumers, signaling potential CPI fluctuations. In the APT model, a rise in WPI may suggest a rise in CPI, impacting expected returns. Data sourced from RBI.

Exchange Rate

USD/INR Exchange Rate reflects the value of the US Dollar against the Indian Rupee. A strong rupee (low rate) may attract foreign investment, potentially boosting stock prices and projected returns in financial modelling. Data was sourced from Yahoo Finance.

Mibor (Mumbai Interbank Offered Rate)

MIBOR in Mumbai is the standard interbank lending rate. A higher MIBOR suggests a tighter monetary policy, possibly leading to decreased economic growth and lower equity returns. Monthly data from FBIL was used (Website: www.fbil.org.in).

Crude Oil Prices for India

Changes in global crude oil prices have significant impacts on various sectors of the economy, such as transportation and energy. Price hikes can raise production costs and lower expected returns, while price drops can benefit certain companies, altering projected returns. The monthly data for Crude Oil Price Basket of India was downloaded from the website of Petroleum planning Analysis Cell (https://ppac.gov.in/).(Indian Basket of Crude is a mix of Oman & Dubai sour grades and Brent sweet grade, based on the ratio of crude processed in the previous financial year. Crude oil prices are average daily prices per month, converted from \$/bbl to INR/bbl for analysis.)

Tools for data collection

1. RBI: (https://cimsdbie.rbi.org.in/DBIE/#/dbie/home)

- 2. Yahoo Finance: (https://finance.yahoo.com/)
- 3. Financial Benchmarks India Pvt. Ltd. (FBIL): (www.fbil.org.in.)
- 4. Petroleum planning Analysis Cell :(https://ppac.gov.in/)

Fama French Four Factor Model Analysis

Monthly returns of mutual fund schemes were studied using data from Dr. Jayant Verma's website, adjusting for survivorship bias. The Fama-French model, including market risk, SMB for size, and HML for value factors, explains stock returns thoroughly. Market risk tracks the S&P 500's performance to compensate for systematic risks. SMB indicates small-cap stocks' historical profitability over large-cap, while HML shows value stocks past success compared to growth stocks. Value stocks are undervalued with low price-to-book ratios, and growth stocks have high valuations due to projected earnings growth. WML, or profitability, considers past returns of successful companies over unsuccessful ones, completing the four-factor model for a comprehensive explanation of stock returns.

Tools for data collection: https://faculty.iima.ac.in/iffm/Indian-Fama-French-Momentum/

DATA ANALYSIS

List of Mutual Fund Schemes under Consideration for Data Analysis Tables 1-11.

	Table 1 LIST OF MUTUAL FUND SCHEMES UNDER CONSIDERATION						
Sr No.	Name Of the Mutual Fund	Ticker (Yahoo Finance)	Inception Date	Market Cap			
1	Nippon India Small Cap Fund - Reg - Growth	0P0001BB5P.BO	Sep 16, 2010	Small Cap			
2	Axis Small Cap Fund - Reg - Growth	0P00011MAV.BO	Dec 05, 2013	Small Cap			
3	ICICI Prudential Small cap Fund - Growth	0P00009QCW.BO	Oct 18, 2007	Small Cap			
4	DSP Small Cap Fund - Reg - Growth	0P0001B9ZV.BO	Jun 14, 2007	Small Cap			
5	HSBC Small Cap Fund - Reg - Growth	0P000133S9.BO	May 12,2014	Small Cap			
6	Sundaram Small Cap Fund - Reg - Growth	0P00005UUL.BO	Feb 15, 2005	Small Cap			
7	Motilal Oswal Midcap Fund - Reg - Growth	0P0001BAYV.BO	Feb 24, 2014	Mid Cap			
8	Nippon India Growth Fund - Reg - Growth	0P0001BB37.BO	Jan 01, 2013	Mid Cap			
9	PGIM India Midcap Opportunities Fund - Reg - Growth	0P0001B9XB.BO	Dec 02, 2013	Mid Cap			
10	Edelweiss Mid Cap Fund - Growth	0P0000A4KC.BO	Jan 01, 2013	Mid Cap			
11	HDFC Mid-Cap Opportunities Fund - Growth	0P00009J3K.BO	Jun 25, 2007	Mid Cap			
12	ICICI Prudential MidCap Fund - Growth	0P00005X04.BO	28-Oct-04	Mid Cap			
13	Baroda BNP Paribas Large Cap Fund - Growth	0P00005V32.BO	Sep 23, 2004	Large Cap			
14	ICICI Prudential Bluechip Fund - Growth	0P0000GB48.BO	May 23,2008	Large Cap			
15	Nippon India Large Cap Fund - Reg - Growth	0P00009JAQ.BO	Aug 08, 2007	Large Cap			
16	Canara Robeco Bluechip Equity Fund - Growth	0P0001B9T5.BO	Aug 20, 2010	Large Cap			
17	HDFC Top 100 Fund - Growth	0P00005WNC.BO	Oct 11, 1996	Large Cap			
18	Bandhan Large Cap Fund - Reg - Growth	0P0001BAKL.BO	Jun 09, 2006	Large Cap			
19	HSBC Large Cap Fund - Growth	0P00005WNT.BO	Jan 01, 2013	Large Cap			
20	SBI Bluechip Fund - Growth	0P00005WEZ.BO	Feb 14, 2006	Large Cap			

Source: Yahoo Finance.

Tools for Data Analysis: Tools to analyse data include descriptive statistics, correlation, and tests of stationarity, OLS regression and Granger causality test, Excel Regression Analysis.

Capm Model Analysis

	Table 2							
~	REGRESSION STATIS	STICS FOR CA	PM MODEL					
Sr No.	Name of Mutual Fund	Beta	P-value	R Square	Adjusted R Square			
1	Nippon India Small Cap Fund - Reg - Growth	1.080	0.000	0.9523	0.9515			
2	Axis Small Cap Fund - Reg - Growth	1.006	0.000	0.9626	0.9620			
3	ICICI Prudential Small cap Fund - Growth	1.077	0.000	0.9508	0.9499			
4	DSP Small Cap Fund - Reg - Growth	1.066	0.000	0.9167	0.9153			
5	HSBC Small Cap Fund - Reg - Growth	1.075	0.000	0.9640	0.9633			
6	Sundaram Small Cap Fund - Reg - Growth	1.105	0.000	0.9557	0.9550			
7	Motilal Oswal Midcap Fund - Reg - Growth	1.029	0.000	0.9187	0.9173			
8	Nippon India Growth Fund - Reg - Growth	1.098	0.000	0.9124	0.9109			
9	PGIM India Midcap Opportunities Fund - Reg -	1.064	0.000	0.9735	0.9730			
	Growth							
10	Edelweiss Mid Cap Fund - Growth	1.043	0.000	0.9772	0.9768			
11	HDFC Mid-Cap Opportunities Fund - Growth	1.030	0.000	0.9797	0.9793			
12	ICICI Prudential Midcap Fund - Growth	1.046	0.000	0.9806	0.9802			
13	Baroda BNP Paribas Large Cap Fund - Growth	0.935	0.000	0.9904	0.9902			
14	ICICI Prudential Bluechip Fund - Growth	0.967	0.000	0.9941	0.9940			
15	Nippon India Large Cap Fund - Reg - Growth	0.935	0.000	0.9650	0.9645			
16	Canara Robeco Bluechip Equity Fund - Growth	0.993	0.000	0.9869	0.9866			
17	HDFC Top 100 Fund - Growth	1.014	0.000	0.9298	0.9286			
18	Bandhan Large Cap Fund - Reg - Growth	0.982	0.000	0.9748	0.9744			
19	HSBC Large Cap Fund - Growth	0.964	0.993	0.9425	0.9929			
20	SBI Bluechip Fund - Growth	0.980	0.000	0.9951	0.9950			

The table shows adjusted R-squared values ranging from 0.9109 to 0.995, indicating varying levels of model accuracy. An adjusted R-squared between 0.9109 and 0.95 implies a good fit for the CAPM model, explaining a significant portion of mutual fund returns linked to the market (Nifty 500). Values of 0.95 to 0.995 signify high model accuracy, explaining a large part of mutual fund return variations tied to the market. A p-value of 0.000 indicates a strong statistical relationship between Nifty 500 and the excess returns of all 20 mutual funds in the model. Overall, the CAPM model offers a good to very good explanation of mutual fund returns variations, but other factors should also be considered for a comprehensive analysis.

The study evaluated beta coefficients in the CAPM model for small, medium, and large capitalization mutual funds. Findings showed that small and mid-cap funds have betas greater than 1.0, indicating higher volatility compared to the market benchmark (Nifty 500). In contrast, large cap funds had a beta around 0.9, suggesting lower volatile returns. These results are consistent with CAPM theory, which suggests that higher risk in small and mid-cap companies leads to potentially higher returns. In

comparison, large-cap companies, with lower risk and growth potential, offer more stable returns. It's important to note that the CAPM model is basic and past performance does not guarantee future results, and beta only measures volatility in relation to the market, not all risk factors.

In Summary

The beta variances witnessed in your CAPM model between small/mid-cap and large-cap funds correspond with what is expected theoretically. Small and mid-cap funds have the possibility of increased returns but also higher risk, while large-cap funds offer a more stable investment option with returns similar to the market average.

Apt (Arbitrage Pricing Theory) Model Analysis

Tools used for Data Analysis: Excel Regression Analysis, Jamovi for PCA and Correlation Matrix. The APT model's data analysis process started with evaluating multicollinearity between the selected variables: GDP, CPI, WPI, crude oil prices, USD/INR exchange rate, and MIBOR rate. Principal Component Analysis (PCA) and Correlation Matrix both involve analysing relationships between variables was performed on the macroeconomic factors under consideration for analysis of data. PCA was conducted to determine fundamental factors that account for the variability in the dataset, whereas correlation matrix was created to analyse the relationships between the variables.

			CORR	ELATIC	Table 3 DN MA'	; TRIX JA]	MOVI						
Correlation Ma	trix												
		MIBO)R	USD/	INR	Crude Pric	e Oil æs	GD	P	СР	I	W	PI
MIBOR	Pearson's r	_											
	p-value	—											
USD/INR	Pearson's r	-0.148											
	p-value	0.247											
Crude Oil Prices	Pearson's r	0.084		0.5421	***								
	p-value	0.513		<.001		_							
GDP	Pearson's r	- 0.1026		0.7943	***	0.7871	***						
	p-value	0.423		<.001		<.001		_					
СРІ	Pearson's r	0.3811	**	0.8974	***	0.6379	***	0.8776	***				
	p-value	0.002		<.001		<.001		<.001		—			
WPI	Pearson's r	- 0.1844		0.8269	***	0.8542	***	0.9491	***	0.9304	***	_	
	p-value	0.148		<.001		<.001		<.001		<.001			
Note. * p < .05	, ** p < .01, *** p <	< .001											

Table 4 PRINCIPAL COMPONENT ANALYSIS JAMOVI				
Component Loadings				
	Component			
	1	Uniqueness		
MIBOR		0.9474		
USD/INR	0.8986	0.1926		
Crude Oil Prices	0.7902	0.3756		
СРІ	0.9634	0.0719		
WPI	0.9513	0.095		
Note. 'varimax' rotation was used				

Examining the PCA components and correlation matrix uncovered the presence of multicollinearity among the variables, especially between WPI and CPI. Multicollinearity arises when two or more independent variables show strong correlations, resulting in regression estimates that are unstable and unreliable.

To tackle this problem, WPI was removed from the model. Although WPI is an indicator of inflation, CPI was kept because it provides a unique view on inflation and how it could affect asset returns. Not including CPI would have led to an inadequate representation of inflationary pressures in the APT analysis.

	Table 5 REGRESSION STATISTICS FOR COEFFICIENTS OF APT MODEL							
				Coefficient Value				
Sr No.	Name of Mutual Fund	MIBOR	USD/INR	Crude Oil Prices in (INR/bbl)	СРІ	GDP At Current Prices (₹ Crore)		
1	Nippon India Small Cap Fund - Reg - Growth	-0.07	-0.01	0.00	0.01	0.00		
2	Axis Small Cap Fund - Reg - Growth	-0.07	-0.01	0.00	0.01	0.00		
3	ICICI Prudential Smallcap Fund - Growth	-0.07	-0.01	0.00	0.01	0.00		
4	DSP Small Cap Fund - Reg - Growth	-0.06	-0.01	0.00	0.01	0.00		
5	HSBC Small Cap Fund - Reg - Growth	-0.07	-0.01	0.00	0.01	0.00		
6	Sundaram Small Cap Fund - Reg - Growth	-0.07	-0.01	0.00	0.01	0.00		
7	Motilal Oswal Midcap Fund - Reg - Growth	-0.07	-0.01	0.00	0.01	0.00		
8	Nippon India Growth Fund - Reg - Growth	-0.07	-0.01	0.00	0.01	0.00		
9	PGIM India Midcap Opportunities Fund - Reg - Growth	-0.07	-0.01	0.00	0.01	0.00		
10	Edelweiss Mid Cap Fund - Growth	-0.07	-0.01	0.00	0.01	0.00		
11	HDFC Mid-Cap Opportunities Fund - Growth	-0.28	-0.01	0.00	0.01	0.00		

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12	ICICI Prudential MidCap Fund -	-0.07	-0.01	0.00	0.01	0.00
	Growth					
13	Baroda BNP Paribas Large Cap	-0.07	-0.01	0.00	0.01	0.00
	Fund - Growth					
14	ICICI Prudential Bluechip Fund -	-0.07	-0.01	0.00	0.01	0.00
	Growth					
15	Nippon India Large Cap Fund - Reg	-0.07	-0.01	0.00	0.01	0.00
	- Growth					
16	Canara Robeco Bluechip Equity	-0.06	-0.01	0.00	0.01	0.00
	Fund - Growth					
17	HDFC Top 100 Fund - Growth	-0.07	-0.01	0.00	0.01	0.00
18	Bandhan Large Cap Fund - Reg -	-0.06	-0.01	0.00	0.01	0.00
	Growth					
19	HSBC Large Cap Fund - Growth	-0.06	-0.01	0.00	0.01	0.00
20	SBI Bluechip Fund - Growth	-0.07	-0.01	0.00	0.01	0.00

The coefficient values given for the APT model indicate that the included factors (GDP, CPI, crude oil prices, exchange rate, and MIBOR rate) show a somewhat limited linear connection with the returns of the 20 mutual funds examined. The MIBOR Beta, Exchange Rate, Crude Oil Prices, CPI, and GDP were analysed for their impact on 20 mutual funds' returns. The MIBOR Beta exhibited a minimal negative impact on profits, while the Exchange Rate showed a slight negative correlation with returns. Crude Oil Prices had no linear connection with fund returns, suggesting inconsistent impacts. A small positive correlation was seen between CPI and returns, implying potential weak influence. GDP growth also showed no linear connection with returns, indicating insignificance. Overall, the selected factors had minimal influence on the fluctuations in mutual funds' returns. The study suggests that these economic indicators may not be major determinants of fund performance.

	Table 6 REGRESSION STATISTICS FOR P- VALUE OF APT MODEL							
				P-value				
Sr No.	Name of Mutual Fund	MIBOR	USD/INR	Crude Oil Prices in (INR/bbl)	СРІ	GDP At Current Prices (₹ Crore)		
1	Nippon India Small Cap Fund - Reg - Growth	0.00	0.12	0.23	0.00	0.01		
2	Axis Small Cap Fund - Reg - Growth	0.00	0.04	0.19	0.00	0.01		
3	ICICI Prudential Smallcap Fund - Growth	0.00	0.06	0.43	0.00	0.00		
4	DSP Small Cap Fund - Reg - Growth	0.00	0.06	0.24	0.00	0.00		
5	HSBC Small Cap Fund - Reg - Growth	0.00	0.07	0.26	0.00	0.01		
6	Sundaram Small Cap Fund - Reg - Growth	0.00	0.11	0.25	0.00	0.01		
7	Motilal Oswal Midcap Fund - Reg - Growth	0.00	0.10	0.63	0.00	0.00		
8	Nippon India Growth Fund - Reg - Growth	0.00	0.23	0.90	0.01	0.01		
9	PGIM India Midcap Opportunities Fund - Reg - Growth	0.00	0.11	0.12	0.00	0.00		

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10	Edelweiss Mid Cap Fund - Growth	0.00	0.09	0.14	0.00	0.01
11	HDFC Mid-Cap Opportunities Fund - Growth	0.00	0.09	0.21	0.00	0.01
12	ICICI Prudential MidCap Fund - Growth	0.00	0.11	0.37	0.00	0.00
13	Baroda BNP Paribas Large Cap Fund - Growth	0.00	0.03	0.05	0.00	0.01
14	ICICI Prudential Bluechip Fund - Growth	0.00	0.08	0.13	0.00	0.01
15	Nippon India Large Cap Fund - Reg - Growth	0.00	0.10	0.36	0.01	0.01
16	Canara Robeco Bluechip Equity Fund - Growth	0.00	0.03	0.09	0.00	0.00
17	HDFC Top 100 Fund - Growth	0.00	0.40	0.72	0.03	0.01
18	Bandhan Large Cap Fund - Reg - Growth	0.00	0.05	0.20	0.00	0.00
19	HSBC Large Cap Fund - Growth	0.00	0.03	0.07	0.00	0.01
20	SBI Bluechip Fund - Growth	0.00	0.07	0.11	0.00	0.01

The p-values given for the coefficients of the APT model reveal information about the statistical importance of each factor's connection to the returns of the 20 mutual funds. Below is an explanation of the significance of each p-value.

Low p-values for MIBOR, CPI, and GDP indicate significant relationships with returns, though the coefficients suggest weak influences. Exchange rates and crude oil prices show varying levels of significance, with potential but inconclusive connections to returns. MIBOR's -0.07 coefficient is highly significant despite a lack of correlation. Exchange rate's -0.01 coefficient may be weakly relevant for certain funds. Crude oil prices with a 0.00 coefficient could be significant for some funds. CPI's 0.01 coefficient implies a weak positive link with returns. GDP's 0.00 coefficient suggests a minimal effect on returns. In conclusion, while some economic indicators show statistical significance, their actual impact on returns may be limited or uncertain. MIBOR and CPI have mild impacts on returns, with negative for MIBOR and slightly positive for CPI. Exchange rates and crude oil prices affect returns differently among funds. Certain factors have statistical connections to returns, but their ability to explain variations is limited. This highlights the need for potential model improvements.

	Table 7								
	REGRESSION STATISTICS FOR R SQUARE & ADJUSTED R SQUARE APT MODEL								
Sr	Name of Mutual Fund	R Square	Adjusted R Square	Standard Error					
No.									
1	Nippon India Small Cap Fund - Reg - Growth	0.76	0.73	0.08					
2	Axis Small Cap Fund - Reg - Growth	0.79	0.77	0.07					
3	ICICI Prudential Smallcap Fund - Growth	0.75	0.73	0.08					
4	DSP Small Cap Fund - Reg - Growth	0.74	0.72	0.08					
5	HSBC Small Cap Fund - Reg - Growth	0.78	0.75	0.08					
6	Sundaram Small Cap Fund - Reg - Growth	0.75	0.72	0.08					
7	Motilal Oswal Midcap Fund - Reg - Growth	0.75	0.73	0.08					
8	Nippon India Growth Fund - Reg - Growth	0.68	0.65	0.10					

9	PGIM India Midcap Opportunities Fund - Reg - Growth	0.80	0.78	0.07
10	Edelweiss Mid Cap Fund - Growth	0.79	0.77	0.07
11	HDFC Mid-Cap Opportunities Fund - Growth	0.78	0.76	0.07
12	ICICI Prudential MidCap Fund - Growth	0.78	0.76	0.07
13	Baroda BNP Paribas Large Cap Fund - Growth	0.84	0.83	0.05
14	ICICI Prudential Bluechip Fund - Growth	0.83	0.81	0.06
15	Nippon India Large Cap Fund - Reg - Growth	0.78	0.76	0.07
16	Canara Robeco Bluechip Equity Fund - Growth	0.82	0.81	0.06
17	HDFC Top 100 Fund - Growth	0.72	0.70	0.08
18	Bandhan Large Cap Fund - Reg - Growth	0.80	0.78	0.07
19	HSBC Large Cap Fund - Growth	0.82	0.80	0.06
20	SBI Bluechip Fund - Growth	0.82	0.80	0.06

The R-squared and adjusted R-squared values of the APT model for 20 mutual funds suggest a good fit between the model and the funds' returns. R-squared (0.755 to 0.84) shows the percentage of variability in returns explained by the model. The adjusted R-squared has improved from 0.733 to 0.83, indicating better reliability for models with multiple variables. A range of 0.733 to 0.83 implies that the model can explain 73.3% to 83% of the return variance, accounting for the factors considered.

Summary of the Analysis

The APT model shown above clarifies a significant portion of the variables that influence the performance of the 20 mutual funds. The model's forecasts match closely with the real returns that were observed.

Fama French Four Factor Model Analysis

The Fama French Four Factor model was analysed using the regression as follows:

	Table 8							
	REGRESSION STATISTICS FOR FAMA FRENCH MODEL COEFFICIENT VALUES							
Sr No.	Name of Mutual Fund		Coefficie	ent Value				
		SMB	HML	WML	Mf-Rf			
1	Nippon India Small Cap Fund - Reg - Growth	0.005	0.002	0.002	1.004			
2	Axis Small Cap Fund - Reg - Growth	0.004	0.000	0.002	0.984			
3	ICICI Prudential Smallcap Fund - Growth	0.006	0.001	0.001	1.008			
4	DSP Small Cap Fund - Reg - Growth	0.008	0.001	0.002	0.973			
5	HSBC Small Cap Fund - Reg - Growth	0.005	0.002	0.002	1.007			
6	Sundaram Small Cap Fund - Reg - Growth	0.006	0.001	0.001	1.030			
7	Motilal Oswal Midcap Fund - Reg - Growth	0.004	0.000	0.000	0.991			
8	Nippon India Growth Fund - Reg - Growth	0.005	0.000	-0.001	1.024			
9	PGIM India Midcap Opportunities Fund - Reg - Growth	0.003	0.000	0.001	1.051			
10	Edelweiss Mid Cap Fund - Growth	0.003	0.000	0.000	1.022			
11	HDFC Mid-Cap Opportunities Fund - Growth	0.003	0.001	0.000	0.984			
12	ICICI Prudential MidCap Fund - Growth	0.003	0.001	0.000	1.002			
13	Baroda BNP Paribas Large Cap Fund - Growth	-0.001	0.000	0.001	0.960			

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14	ICICI Prudential Bluechip Fund - Growth	-0.001	0.001	0.000	0.972
15	Nippon India Large Cap Fund - Reg - Growth	0.000	0.001	0.000	0.966
16	Canara Robeco Bluechip Equity Fund - Growth	-0.001	-0.001	0.001	0.970
17	HDFC Top 100 Fund - Growth	0.000	0.002	-0.001	0.970
18	Bandhan Large Cap Fund - Reg - Growth	0.000	-0.001	0.001	1.001
19	HSBC Large Cap Fund - Growth	-0.001	-0.001	0.000	0.987
20	SBI Bluechip Fund - Growth	-0.001	0.000	0.000	0.983

The coefficients for the Fama-French Four-Factor Model on 20 mutual funds show a slight to minimal correlation between SMB, HML, WML factors and returns, with market risk as the dominant factor. SMB range of -0.001 to 0.008 suggests minimal size impact. HML range of 0.001 to 0.007 shows minimal value factor effect. WML range of -0.001 to 0.002 indicates low momentum impact. Market Risk of 0.970 to 1.004 shows a strong positive correlation with portfolio returns, closely following market movements. The analysis suggests small changes in factors have minimal impact on portfolio performance. Market risk is the primary factor affecting the returns. This indicates that the mutual funds' performance is mainly influenced by overall market trends rather than individual company sizes, value traits, or previous performance.

Table 9 REGRESSION STATISTICS FOR FAMA FRENCH MODEL P VALUES								
Sr No.	Name of Mutual Fund	P-value						
		SMB	HML	WML	Mf-Rf			
1	Nippon India Small Cap Fund - Reg - Growth	0.000	0.012	0.018	0.000			
2	Axis Small Cap Fund - Reg - Growth	0.000	0.548	0.010	0.000			
3	ICICI Prudential Smallcap Fund - Growth	0.000	0.362	0.292	0.000			
4	DSP Small Cap Fund - Reg - Growth	0.000	0.178	0.078	0.000			
5	HSBC Small Cap Fund - Reg - Growth	0.000	0.008	0.017	0.000			
6	Sundaram Small Cap Fund - Reg - Growth	0.000	0.327	0.274	0.000			
7	Motilal Oswal Midcap Fund - Reg - Growth	0.007	0.967	0.723	0.000			
8	Nippon India Growth Fund - Reg - Growth	0.001	0.779	0.448	0.000			
9	PGIM India Midcap Opportunities Fund - Reg - Growth	0.000	0.714	0.046	0.000			
10	Edelweiss Mid Cap Fund - Growth	0.000	0.484	0.504	0.000			
11	HDFC Mid-Cap Opportunities Fund - Growth	0.000	0.129	0.655	0.000			
12	ICICI Prudential MidCap Fund - Growth	0.000	0.141	0.479	0.000			
13	Baroda BNP Paribas Large Cap Fund - Growth	0.023	0.477	0.026	0.000			
14	ICICI Prudential Bluechip Fund - Growth	0.000	0.031	0.510	0.000			
15	Nippon India Large Cap Fund - Reg - Growth	0.687	0.007	0.295	0.000			
16	Canara Robeco Bluechip Equity Fund - Growth	0.181	0.263	0.486	0.000			
17	HDFC Top 100 Fund - Growth	0.840	0.153	0.284	0.000			
18	Bandhan Large Cap Fund - Reg - Growth	0.810	0.258	0.417	0.000			
19	HSBC Large Cap Fund - Growth	0.043	0.021	0.677	0.000			
20	SBI Bluechip Fund - Growth	0.031	0.696	0.453	0.000			

General Analysis

The p-values confirm the results of the coefficient analysis. The factors of size (SMB), value (HML), and momentum (WML) are unlikely to have a strong impact on the returns of the majority of funds, and this impact may not be statistically significant in every instance. Market risk continues to be the primary factor affecting all funds, showing a notable impact in statistical terms. The Fama-French Four-Factor Model is not very effective in explaining the fluctuations in returns of the majority of the 20 mutual funds analysed in this case. While a few funds may demonstrate limited correlations with size, value, or momentum, the majority of funds do not have statistically significant relationships with these factors. The main factor affecting returns is still market risk.

Table 10 REGRESSION STATISTICS FOR FAMA FRENCH MODEL R SQUARE AND ADJUSTED R SQUARE								
Sr No.	VA Name of Mutual Fund	<u>LUES</u> R Square	Adjusted R Square	Standard Error				
1	Nippon India Small Cap Fund - Reg - Growth	0.976	0.975	0.025				
2	Axis Small Cap Fund - Reg - Growth	0.979	0.978	0.021				
3	ICICI Prudential Smallcap Fund - Growth	0.974	0.973	0.026				
4	DSP Small Cap Fund - Reg - Growth	0.960	0.957	0.032				
5	HSBC Small Cap Fund - Reg - Growth	0.983	0.982	0.021				
6	Sundaram Small Cap Fund - Reg - Growth	0.982	0.981	0.022				
7	Motilal Oswal Midcap Fund - Reg - Growth	0.929	0.924	0.041				
8	Nippon India Growth Fund - Reg - Growth	0.932	0.927	0.043				
9	PGIM India Midcap Opportunities Fund - Reg - Growth	0.980	0.979	0.022				
10	Edelweiss Mid Cap Fund - Growth	0.983	0.982	0.020				
11	HDFC Mid-Cap Opportunities Fund - Growth	0.988	0.987	0.017				
12	ICICI Prudential MidCap Fund - Growth	0.988	0.988	0.016				
13	Baroda BNP Paribas Large Cap Fund - Growth	0.993	0.992	0.011				
14	ICICI Prudential Bluechip Fund - Growth	0.996	0.995	0.009				
15	Nippon India Large Cap Fund - Reg - Growth	0.990	0.989	0.015				
16	Canara Robeco Bluechip Equity Fund - Growth	0.969	0.967	0.024				
17	HDFC Top 100 Fund - Growth	0.937	0.933	0.038				
18	Bandhan Large Cap Fund - Reg - Growth	0.976	0.975	0.022				
19	HSBC Large Cap Fund - Growth	0.995	0.994	0.010				
20	SBI Bluechip Fund - Growth	0.996	0.995	0.010				

High R-squared and adjusted R-squared values when using the Fama-French Four-Factor Model on 20 mutual funds indicate a strong relationship between the model and fund performance. R-squared shows the percentage of return variability explained by the model's factors, ranging from 93% to 100%. Adjusted R-squared penalizes unnecessary variables, resulting in a more reliable assessment of the model. With values from 0.92 to 1, it suggests that 92% to 100% of return variability is clarified by the model. Overall, the model effectively explains factors impacting the funds' performance, with predictions closely matching observed returns.

Model Comparison Analysis

For further analysis on the research topic the Adjusted R Square values obtained from all the three models are compared to each other to determine which model fits bets to the Indian Open ended equity Mutual fund schemes. Out of the three models, the Fama-French Four-Factor Model has the highest adjusted R-squared, suggesting the strongest fit. This study shows that by taking into account variables including size, value, market risk, and momentum, the Fama-French model accurately predicts the variations in mutual fund returns for Indian open-ended equity schemes. Subsequent examination of the Capital Asset Pricing Model's (CAPM) adjusted R-squared value revealed that the APT and Fama-French models is less accurate than the CAPM model, even though the Fama-French model offers a more thorough explanation for return variability by include other factors outside market risk. Ultimately, the APT model's adjusted R-squared value was the lowest, suggesting that it was less accurate than the other two models. This demonstrates that, in comparison to the other two models, the variables assessed by APT—which are derived from statistics rather than preset—are less successful in understanding the fluctuations in returns of these Indian mutual funds.

Table 11 REGRESSION STATISTICS ADJUSTED R SQUARE VALUES								
Sr No.	Name of Mutual Fund	CAPM	APT	Fama French				
		Adjusted R Square						
1	Nippon India Small Cap Fund - Reg - Growth	0.95	0.73	0.97				
2	Axis Small Cap Fund - Reg - Growth	0.96	0.77	0.98				
3	ICICI Prudential Smallcap Fund - Growth	0.95	0.73	0.97				
4	DSP Small Cap Fund - Reg - Growth	0.92	0.72	0.96				
5	HSBC Small Cap Fund - Reg - Growth	0.96	0.75	0.98				
6	Sundaram Small Cap Fund - Reg - Growth	0.95	0.72	0.98				
7	Motilal Oswal Midcap Fund - Reg - Growth	0.92	0.73	0.92				
8	Nippon India Growth Fund - Reg - Growth	0.91	0.65	0.93				
9	PGIM India Midcap Opportunities Fund - Reg - Growth	0.97	0.78	0.98				
10	Edelweiss Mid Cap Fund - Growth	0.98	0.77	0.98				
11	HDFC Mid-Cap Opportunities Fund - Growth	0.98	0.76	0.99				
12	ICICI Prudential MidCap Fund - Growth	0.98	0.76	0.99				
13	Baroda BNP Paribas Large Cap Fund - Growth	0.99	0.83	0.99				
14	ICICI Prudential Bluechip Fund - Growth	0.99	0.81	1.00				
15	Nippon India Large Cap Fund - Reg - Growth	0.96	0.76	0.99				
16	Canara Robeco Bluechip Equity Fund - Growth	0.99	0.81	0.97				
17	HDFC Top 100 Fund - Growth	0.93	0.70	0.93				
18	Bandhan Large Cap Fund - Reg - Growth	0.97	0.78	0.97				
19	HSBC Large Cap Fund - Growth	0.99	0.80	0.99				
20	SBI Bluechip Fund - Growth	1.00	0.80	1.00				

CONCLUSION

Results from the study confirmed the first hypothesis that the Fama-French Four-Factor Model provides a more comprehensive explanation for the risk-return trade-off in Indian open-ended equities mutual funds compared to the CAPM and APT models. The Fama-French model displayed the highest adjusted R-squared, indicating a better fit for the data. However, there are still unexplained variations in return variability demonstrated by all three models producing adjusted R-squared values less than 1. The APT model, in particular, showed a poor match, suggesting its generated factors may not be significant. The study suggests that multi-factor models like Fama-French may be advantageous in analysing risk and return connections in Indian equities due to factors beyond market risk.

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